I

a) $\cos 2x + 5\cos x + 3 = 0$

$\cos^2 x - \sin^2 x +$

$2\cos^2 x - 1 + 5\cos x + 3 = 0$

$2\cos^2 x + 5\cos x + 2 = 0$

$(2\cos x + 1)(\cos x + 2)$

$\cos x = -\frac{1}{2}$ $\quad$ $\cos x > -2$ illegal

$x \in \{\frac{2\pi}{3}, \frac{4\pi}{3}\}$

b) $\sin 2x \tan^2 2x - \tan 2x = \sin 2x$

$\sin 2x \left(\frac{\sin^2 2x}{\cos^2 2x} - \frac{1}{\cos 2x} - 1\right) = 0$

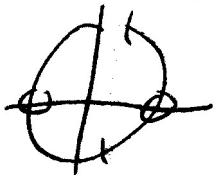
$\frac{\sin 2x}{\cos^2 2x}\left(\sin^2 2x - \cos 2x - \cos^2 2x\right) = 0$

$1 - \cos^2 2x$

$-\frac{\sin 2x}{\cos^2 2x}\left(2\cos^2 2x + \cos 2x - 1\right) = 0$

$-\frac{\sin 2x}{\cos^2 2x}(2\cos 2x - 1)(\cos 2x + 1)$

$\cos 2x = \frac{1}{2}$ $\quad$ $\cos 2x = -1$



$2x \in \{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi, \frac{7\pi}{3}, 3\pi, \frac{11\pi}{3}\}$

$x \in \{0, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\}$

c) $\cos^2 3x + \sin 3x = \frac{5}{4}$

$1 - \sin^2 3x$

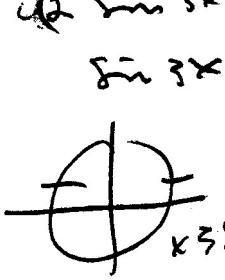
$4 - 4\sin^2 3x + 4\sin 3x = 5$

$4\sin^2 3x - 4\sin 3x + 1 = 0$

$(2\sin 3x - 1)^2 = 0$

$\sin 3x = \frac{1}{2}$

$3x \in \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}\}$



$x \in \{\frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}\}$

D) $\tan 2x + 2\cos x = 0$ $\quad$ $\div 2\cos$

$\frac{\sin 2x}{\cos 2x} + 2\cos x = 0$

$\frac{2\sin x\cos x}{\cos^2 x - \sin^2 x} + 2\cos x = 0$

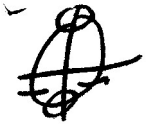
$2\cos x\left[\frac{\sin x}{1 - 2\sin^2 x} + 1\right] = 0$

$\frac{2\cos x}{1 - 2\sin^2 x}\left[\sin x + 1 - 2\sin^2 x\right] = 0$

$\frac{-2\cos x}{1 - 2\sin^2 x}\left[2\sin^2 x - \sin x - 1\right] = 0$

$\frac{-2\cos x}{1 - 2\sin^2 x}(2\sin x + 1)(\sin x - 1)$

$\sin x = -\frac{1}{2}$ $\quad$ $\sin x = 1$



$x \in \{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\}$

E) $\tan^2 3x = 1$

$\tan 3x = \pm 1$

4 pts

$3x \in \{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}, \frac{15\pi}{4}, \frac{17\pi}{4}, \frac{19\pi}{4}, \frac{21\pi}{4}, \frac{23\pi}{4}\}$

$x \in \{\frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}, \frac{7\pi}{4}, \frac{3\pi}{4}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{5\pi}{4}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{7\pi}{4}, \frac{23\pi}{12}\}$

II

#1 $\cot^{-1} x = \tan^{-1}\left(\frac{1}{x}\right) + \pi \quad \forall x<0$

let $\cot^{-1} x = b \qquad x<0 \qquad \frac{\pi}{2} < b < \pi$ since $x<0$

$\cot \cot^{-1} x = \cot b$

$x = \cot b$

$\frac{1}{x} = \tan b$

$\tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}(\tan b)$

$= \tan^{-1}(\tan(b-\pi))$

$\tan^{-1}\frac{1}{x} = b - \pi$

$\tan^{-1}\frac{1}{x} + \pi = b$

$\tan^{-1}\frac{1}{x} + \pi = \cot^{-1} x$

Range of $\tan^{-1} u \quad -\frac{\pi}{2} < u < \frac{\pi}{2}$

in this case $-\frac{\pi}{2} < u < 0$

$\tan^{-1}\tan b \neq b$ because b is in 2nd Quad,

$\tan(b-\pi) \overset{?}{=} \tan b$

$\frac{\tan b - \tan \pi}{1 + \tan b \tan \pi} = \frac{\tan b - 0}{1 - 0} = \tan b.$

so $\tan b = \tan(b-\pi)$

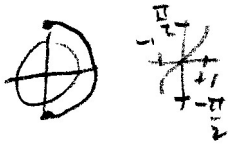
and $-\frac{\pi}{2} < b - \pi < 0$

III

A) $A(x) = \sin^{-1} x$

$D_A \quad -1 \le x \le 1$

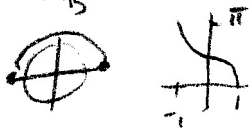
$R_A \quad -\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$



B) $B(x) = \cos^{-1} x$

$D_B \quad -1 \le x \le 1$

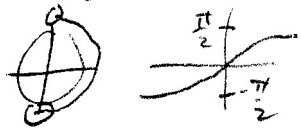
$R_B \quad 0 \le \cos^{-1} x \le \pi$



C) $C(x) = \tan^{-1} x$

$D_C \quad x \in \mathbb{R}$

$R_C \quad -\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$



D) $D(x) = \cot^{-1} x$

$D_D \quad x \in \mathbb{R}$

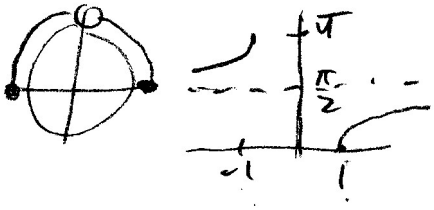
$R_D \quad 0 < \cot^{-1} x < \pi$

E) $E(x) = \sec^{-1} x$

$D_E \quad x \le -1$ or $x \ge 1$

$R_E \quad 0 \le \sec^{-1} x \le \pi;$

$\sec^{-1} x \neq \frac{\pi}{2}$

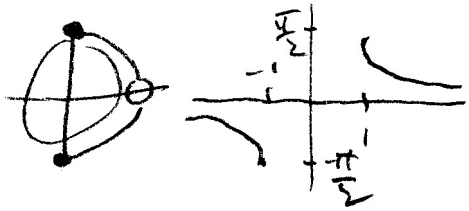


F) $F(x) = \csc^{-1} x$

$D_F \quad x \le -1$ or $x \ge 1$

$R_F \quad -\frac{\pi}{2} \le \csc^{-1} x \le \frac{\pi}{2};$

$\csc^{-1} x \neq 0$



Extra

$\sin x \cos x = \frac{3}{4}$ so

$2 \sin x \cos x = \frac{3}{2}$

$\sin 2x = \frac{3}{2}$

Reject

Range of $\sin 2x$ is between -1 and 1 and $\frac{3}{2}$ is outside that range so $\sin 2x$ never $= \frac{3}{2}$

so $\sin x \cos x$ never $= \frac{3}{4}$,

11/20/2000

10	1	2	3
90	4	6	10
80	4	4	8
70	3	4	7
60	3	5	8
59	7	4	11

105

$\frac{1510}{21}$ $\frac{1998}{25}$ $\frac{3508}{23}$

71.9 79.9 76.3

Replace I d

because solutions $\frac{\pi}{2}$, $\frac{3\pi}{2}$

create havoc w/in the solving of the equations.

Put in a "~~sin add~~ cosine add pblm

like ~~#~~ sample #5 of lecture on Day 1 of freedom

Honors Trigonometry Name TRg ch 4

Copy original problem. Per _____ Date 2000

Convince *me* that **you** understand the concept!

No Calculators, of course.

Chapter 4 Exam

I

Solve $\forall\ x\ \varepsilon\ [0,2\pi)$ (i.e. $0 \le x < 2\pi$).
Use proper set notation and include "circle solutions" where appropriate. (15 pts ea.)

a) $\cos 2x + 5\cos x + 3 = 0$

b) $\sin 2x \tan^2 2x - \tan 2x = \sin 2x$

Final

c) $\cos^2 3x + \sin 3x = \frac{5}{4}$

d) $\tan 2x + 2\cos x = 0$

e) $\tan^2 3x = 1$

Final

II

Prove $\cot^{-1} x = \tan^{-1}\left(\frac{1}{x}\right) + \pi$ for $x < 0$ (15 pts)

Include explicit comments on each line explaining or justifying what you've done.

III

For each of the following functions, state the domain and range, draw the graph of the function on a ***properly labeled axis*** and show the "memory device" we use for the function. (tot 10 pts)

a) $A(x) = \sin^{-1} x$ b) $B(x) = \cos^{-1} x$ c) $C(x) = \tan^{-1} x$

d) $D(x) = \cot^{-1} x$ e) $E(x) = \sec^{-1} x$ f) $F(x) = \csc^{-1} x$

Extra Credit ---------- 5 pts ----------

Final

Show that there is no real number x such that $\sin x \cos x = \frac{3}{4}$. Be *very* convincing.

I

$\cos^{-1}x + \sin^{-1}x = k$

$\cos\cos^{-1}x\cos\sin^{-1}x - \sin\cos^{-1}x\sin\sin^{-1}x = \cos k$

$x\sqrt{1-x^2} - \sqrt{1-x^2}\,(x) = \cos k$

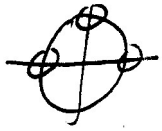
$0 = \cos k$

$\therefore k = \frac{\pi}{2}$

II

A) $\sin 2x + \sec^2 2x = 1 + \sin^2 2x + \tan^2 2x.$

$\sin 2x = \sin^2 2x$



$\sin 2x(\sin 2x - 1) = 0$

$2x \in \{0, \frac{\pi}{2}, \pi, 2\pi, \frac{5\pi}{2}, 3\pi\}$

$x \in \{0, \frac{\pi}{4}, \frac{\pi}{2}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}\}$

Reject ± 0

$x \in \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\}$

B) $\cos 2x + 2\sin^2\frac{x}{2} = 1$

$\cos^2 x - \sin^2 x + 2\sin^2\frac{x}{2} = 1$

$-\sin^2 x + 2\sin^2\frac{x}{2} = 1 - \cos^2 x$

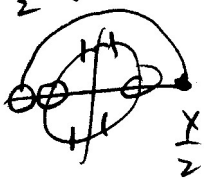
$= \sin^2 x$

$-\cancel{2}\sin^2 x + \cancel{2}\sin^2\frac{x}{2} = 0$

$-4\sin^2\frac{x}{2}\cos^2\frac{x}{2} + \sin^2\frac{x}{2} = 0$

$\sin^2\frac{x}{2}(4\cos^2\frac{x}{2} + 1) = 0$

$\cos\frac{x}{2} = \pm\frac{1}{2}$



$\frac{x}{2} \in \{0, \frac{\pi}{3}, \frac{2\pi}{3}\}$

$x \in \{0, \frac{2\pi}{3}, \frac{4\pi}{3}\}$

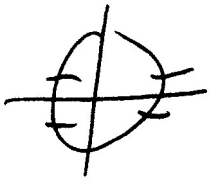
c) $\tan 2x \tan x = 1$

$\frac{2\tan^2 x}{1 - \tan^2 x} = 1$

$2\tan^2 x = 1 - \tan^2 x$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm\frac{1}{\sqrt{3}}$



$x \in \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

d) $\sin 2x + \cos 2x - 1 = 0$

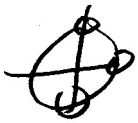
$\sin 2x = 1 - \cos 2x$

$\sin^2 2x = 1 - 2\cos 2x + \cos^2 2x$

$1 - \cos^2 2x$

$2\cos^2 2x - 2\cos 2x = 0$

$2\cos 2x(\cos 2x - 1) = 0$



$x \in \{0, \frac{\pi}{2}, \frac{3\pi}{2}\}$

x	
0	$0+1+1=2$ ok
$\frac{\pi}{2}$	$1+0-1=0$ ok
$\frac{3\pi}{2}$	$-1+0-1=0$ NO
π	$1+0-1=0$

$x \in \{0, \frac{\pi}{2}\}$

III

$$\cot^{-1}x = \begin{cases} \tan^{-1}\frac{1}{x} + \pi & \text{for } x<0 \\ \tan^{-1}\frac{1}{x} & \text{for } x>0 \end{cases}$$

If $x<0$

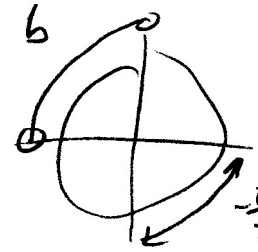
let $b = \cot^{-1}x$ $\quad 0<b<\pi$

but when $x<0$, then

$\frac{\pi}{2} < b < \pi$

$\cot b = x$

$\tan b = \frac{1}{x}$



$-\frac{\pi}{2} < \tan^{-1}x < 0$

when $x<0$

since tangent has period π then $\tan b = \tan(b+\pi) = \tan(b-\pi)$

$\frac{\pi}{2}<b<\pi \quad \frac{3\pi}{2}, \frac{3\pi}{2} \quad -\frac{\pi}{2} \quad \frac{\pi}{2}$

$$\tan(b-\pi) = \frac{1}{x}$$
$$\tan^{-1}\tan(b-\pi) = \tan^{-1}\frac{1}{x}$$
$$b-\pi = \tan^{-1}\frac{1}{x}$$
$$b = \tan^{-1}\frac{1}{x} + \pi$$
$$\cot^{-1}x = \tan^{-1}\frac{1}{x} + \pi$$

If $x>0$

let $b = \cot^{-1}x \quad 0<b<\frac{\pi}{2}$ since $x>0$

$$\cot b = \cot(\cot^{-1}x)$$
$$= x$$
$$\tan b = \frac{1}{x}$$
$$b = \tan^{-1}\frac{1}{x}$$
$$\cot^{-1}x = \tan^{-1}\frac{1}{x}$$

12/29/2000

Sunday Sonkaron 81
Stephanie Kurtz 39

Honors Trigonometry

Name Trg Ch 4

Copy original problem.

Per _____ Date MU

Convince *me* that **you** understand the concept!

No Calculators!

Chapter 4 Make Up Exam

I Range Domain Ia of 6 Inverse Trig Ratios 20 pts

I Given the constant k such that $0 \le k \le \pi$ and $\cos^{-1} x + \sin^{-1} x = k$.

Determine the exact value of k. (~~20~~ 15 pts)

II Solve $\forall\ x\ \varepsilon\ [0, 2\pi)$ (i.e. $0 \le x < 2\pi$).
Use proper set notation and include "circle solutions" where appropriate. (15 pts ea.)

~~a)~~ $\sin 2x + \sec^2 2x = 1 + \sin^2 2x + \tan^2 2x$ #28

a b) $\cos 2x + 2\sin^2 \frac{x}{2} = 1$ #51

b ~~c)~~ $\tan 2x \tan x = 1$

c d) $\sin x + \cos x - 1 = 0$

III ~~Prove~~ Convince: $\cot^{-1} x = \begin{cases} \tan^{-1} \frac{1}{x} + \pi & \text{for } x < 0 \\ \tan^{-1} \frac{1}{x} & \text{for } x > 0 \end{cases}$ (20 pts)

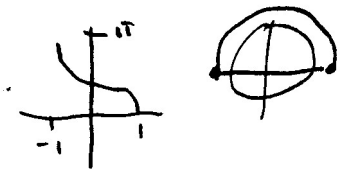
a) Prove the $x > 0$ case ~~10~~ 15 pts
b) Prove the $x < 0$ case ~~10~~ 5 pts
Extra.

I

a) $a(x) = \cos^{-1} x$

$D_a \quad -1 \le x \le 1$

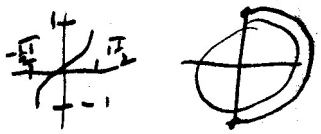
$R_a \quad 0 \le a(x) \le \pi$



b) $b(x) = \sin^{-1} x$

$D_b \quad -1 \le x \le 1$

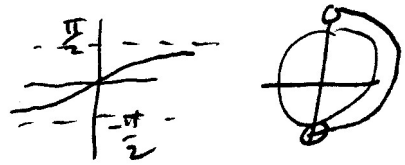
$R_b \quad -\frac{\pi}{2} \le b(x) \le \frac{\pi}{2}$



c) $c(x) = \tan^{-1} x$

$D_c \quad x \in \mathbb{R}$

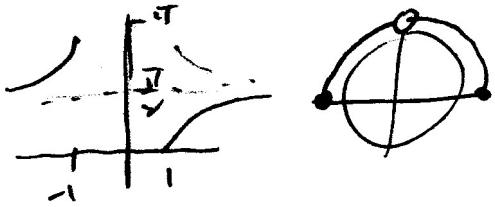
$R_c \quad -\frac{\pi}{2} < c(x) < \frac{\pi}{2}$



d) $d(x) = \sec^{-1} x$

$D_d \quad x \le -1$ or $x \ge 1$

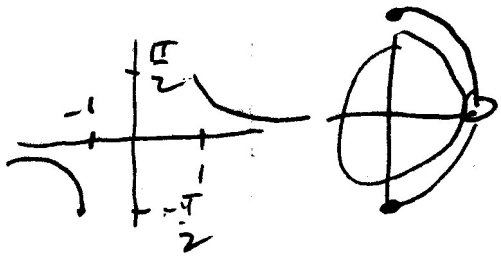
$R_d \quad 0 \le d(x) \le \pi,$
$d(x) \ne \frac{\pi}{2}$



e) $e(x) = \csc^{-1} x$

$D_e \quad x \le -1$ or $x \ge 1$

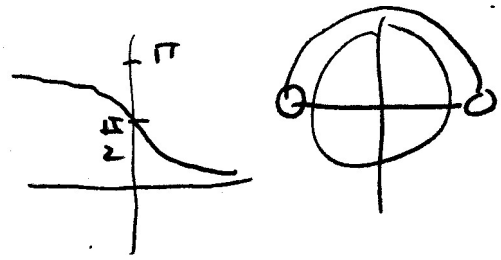
$R_e \quad -\frac{\pi}{2} < e(x) \le \frac{\pi}{2},$
$e(x) \ne 0$



f) $f(x) = \cot^{-1}(x)$

$D_f \quad x \in \mathbb{R}$

$R_f \quad 0 < f(x) < \pi$



II a) $\tan 2x = \tan x$

$$\frac{2\tan x}{1-\tan^2 x} - \tan x = 0$$

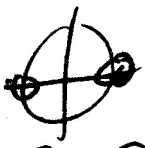
$$2\tan x - \tan x + \tan^3 x = 0$$

$$\tan x + \tan^3 x = 0$$

$$\tan x\,(1 + \tan^2 x) = 0$$

Reject. 2pts

$$\tan x = 0$$



$x \in \{0, \pi\}$

OR

$$\frac{\sin 2x}{\cos 2x} = \frac{\sin x}{\cos x}$$

$$\sin 2x \cos x = \sin x \cos 2x$$

$$\sin 2x \cos x - \sin x \cos 2x = 0$$

$$\sin(2x - x) = 0$$

b) $$\frac{\sin 2x}{1+\cos 2x} = \tan x$$

$$\frac{2\sin x\cos x}{1+\cos^2 x - \sin^2 x} - \frac{\sin x}{\cos x} = 0$$

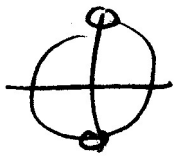
$$\frac{\cancel{2}\sin x\cos x}{\cancel{2}\cos^2 x} - \frac{\sin x}{\cos x} = 0$$

$$\sin x\left(\frac{1-1}{\cos x}\right) = 0$$

$$\sin x \cdot 0 = 0$$

$$0 = 0$$

Identity.



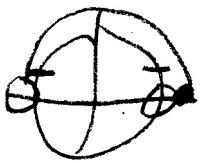
$0 \le x \le 2\pi,\ x \notin \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$ 3pts

II c) $\sin\frac{x}{2} + \cos x = 1$

$\sin\frac{x}{2} + \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = 1$

$\sin\frac{x}{2} + \cancel{1} - 2\sin^2\frac{x}{2} = \cancel{1}$

$-\sin\frac{x}{2}\left(2\sin\frac{x}{2} - 1\right) = 0$



$\sin\frac{x}{2} = \frac{1}{2}$

$\frac{x}{2} \in \left\{0, \frac{\pi}{6}, \frac{5\pi}{6}\right\}$

No $\frac{x}{2} + \frac{x}{2}$ -10

$x \in \left\{0, \frac{\pi}{3}, \frac{5\pi}{3}\right\}$

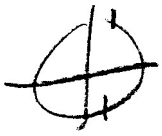
E) $\sin 2x < \sin x$

$2\sin x\cos x - \sin x < 0$

$\sin x\,(2\cos x - 1) < 0$

$0 \quad + \quad 0 \quad - \quad 0 \quad + \quad 0 \quad - \quad 0$

$0 \quad \frac{\pi}{3} \quad \frac{\pi}{2} \quad \pi \quad \frac{3\pi}{2} \quad \frac{5\pi}{3} \quad 2\pi$



No #lin -5

and wrong answer -8

$\frac{\pi}{2} \to 1(-1) < 0$

$\frac{\pi}{3} < x < \pi$ or $\frac{5\pi}{3} < x < 2\pi$

Extra

$4\sin x\cos x = 1$

$2\sin 2x = 1$

$\sin 2x = \frac{1}{2}$

$0 \le x < 2\pi$

$0 \le 2x < 4\pi$

d) $\frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = 4$

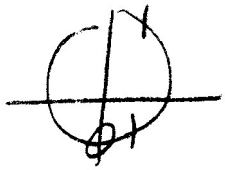
$1 + 2\sin x + \sin^2 x + \cos^2 x = 4\cos x + 4\sin x\cos x$

$2 + 2\sin x = 4\cos x + 4\sin x\cos x$

$2\left[1 + \sin x - 2\cos x(1 + \sin x)\right] = 0$

$2\left[(1+\sin x)(1 - 2\cos x)\right] = 0$

$\sin x = -1 \qquad \cos x = \frac{1}{2}$



$x \in \left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

Reject. 2pts

III $\tan^{-1}\frac{4}{7} + \tan^{-1}x = \frac{\pi}{2}$

cos(

$\cos(\tan^{-1}\frac{4}{7})\cos(\tan^{-1}x) - \sin(\tan^{-1}\frac{4}{7})\sin(\tan^{-1}x) = 0$

$\frac{1}{\sqrt{1+\frac{16}{49}}} \cdot \frac{1}{\sqrt{1+x^2}} - \frac{\frac{4}{7}}{\sqrt{1+\frac{16}{49}}} \cdot \frac{x}{\sqrt{1+\frac{16}{49}}} = 0$

$1 - \frac{4}{7}x = 0$

$x = \frac{7}{4}$

"Reject" $= -6$ with explanation.

11/16/2001

	3	5	T
100			
90	4	4	
80	5	6	
70	6	8	
60	3	5	
59	5	5	

94

23

$\frac{1628}{22}$ $\frac{2066}{28}$

74.0 73.8 73.9

a disappointing showing.
I expected 5 pts higher.

Excellent exam. No changes.

97

100↑	0	
90↑	2	
80↑	8	
70↑	8	
60↑	1	
59↓	3	

96

74

29

$\frac{1635}{22}$

make ID the ex
and toss the ex
might have been
tiny bit too long
w/out the bottom 2
average grade

Honors Trigonometry

Copy original problem.

Convince *me* that **you** understand the concept*!*

No Calculators!

Name Trig Ch 4

Per _____ Date ~~2001~~

Chapter 4 Exam

I State the Domain and Range, show a sketch of the function and show the "memory device" for each of the following: (15 pts tot)

a) $a(x) = \cos^{-1} x$ b) $b(x) = \sin^{-1} x$ c) $c(x) = \tan^{-1} x$

d) $d(x) = \sec^{-1} x$ e) $e(x) = \csc^{-1} x$ f) $f(x) = \cot^{-1} x$

II Solve $\forall\ x$ such that $0 \le x < 2\pi$ (15 pts ea)

a) $\tan 2x = \tan x$

b) $\dfrac{\sin 2x}{1 + \cos 2x} = \tan x$

Fall A

c) $\sin \dfrac{x}{2} + \cos x = 1$

Fall B

d) $\dfrac{1 + \sin x}{\cos x} + \dfrac{\cos x}{1 + \sin x} = 4$

with final

e) $\sin 2x < \sin x$

III Solve $\forall\ x\ \varepsilon\ \Re$ $\tan^{-1} \dfrac{4}{7} + \tan^{-1} x = \dfrac{\pi}{2}$ (10 pts)

Extra Credit ---------- **5 pts** ----------

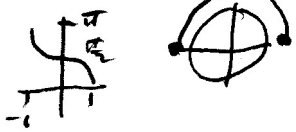
Solve $\forall\ x\ \ 0 \le x < 2\pi$: $4 \sin x \cos x = 1$

I

a) $a(x) = \cos^{-1} x$

D_a $-1 \le x \le 1$

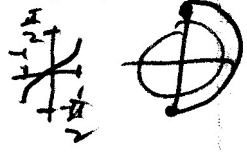
R_a $0 \le \cos^{-1} x \le \pi$



b) $b(x) = \sin^{-1} x$

D_b $-1 \le x \le 1$

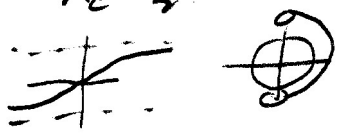
R_b $-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$



c) $c(x) = \tan^{-1}(x)$

D_c $x \in \mathbb{R}$

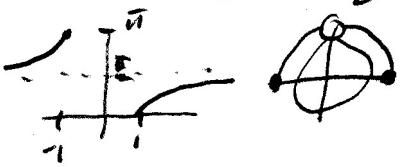
R_c $-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$



d) $d(x) = \sec^{-1} x$

D_d $x \le -1$ or $x \ge 1$

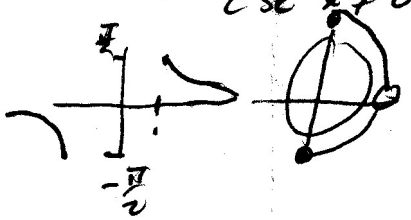
R_d $0 \le \sec^{-1} x \le \pi$; $\sec^{-1} x \ne \frac{\pi}{2}$



e) $e(x) = \csc^{-1} x$

D_e $x \le -1$ or $x \ge 1$

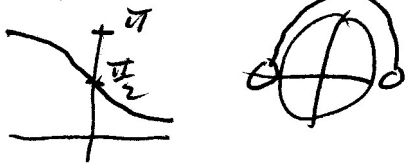
R_e $-\frac{\pi}{2} \le \csc^{-1} x \le \frac{\pi}{2}$; $\csc^{-1} x \ne 0$



$f(x) = \cot^{-1} x$

D_f $x \in \mathbb{R}$

R_f $0 < \cot^{-1} x < \pi$



II $\cos^{-1} x + \sin^{-1} x = K$ $\quad 0 \le K \le \pi$

$\cos(\cos^{-1} x + \sin^{-1} x) = \cos K$

$\cos \cos^{-1} x \cos \sin^{-1} x - \sin \cos^{-1} x \sin \sin^{-1} x = \cos K$

$x\sqrt{1-x^2} - (\sqrt{1-x^2})\, x = \cos K$

$0 = \cos K$

$K = \frac{\pi}{2}$

$\cos(\cos^{-1} x) + \sin(\sin^{-1} x) = K$ -12

III

A) $\cos 2x + 2\sin^2 \frac{x}{2} = 1$

becomes $x = -10$

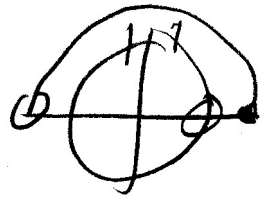
$0 \le x < 2\pi$

$0 \le \frac{x}{2} < \pi$

$\cos^2 x - \sin^2 x + 2\sin^2 \frac{x}{2} = 1$

$-\sin^2 x + 2\sin^2 \frac{x}{2} = 1 - \cos^2 x$

$= \sin^2 x$



$-\cancel{2}\sin^2 x + \cancel{2}\sin^2 \frac{x}{2} = 0$

$-(2\sin\frac{x}{2}\cos\frac{x}{2})^2 + \sin^2\frac{x}{2} = 0$

$-4\sin^2\frac{x}{2}\cos^2\frac{x}{2} + \sin^2\frac{x}{2} = 0$

$\sin^2\frac{x}{2}(-4\cos^2\frac{x}{2} + 1) = 0$

$\sin^2\frac{x}{2} = 0$ or $\cos^2\frac{x}{2} = \frac{1}{4}$

$\cos\frac{x}{2} = \pm\frac{1}{2}$

$\frac{x}{2} = \{0, \frac{\pi}{3}, \frac{2\pi}{3}\}$

$x = \{0, \frac{2\pi}{3}, \frac{4\pi}{3}\}$.

III

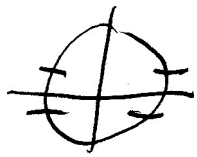
b) $\tan 2x \tan x = 1$

$\frac{2\tan^2 x}{1-\tan^2 x} = 1$

$2\tan^2 x = 1 - \tan^2 x$

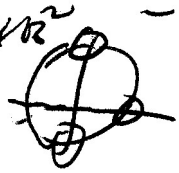
$3\tan^2 x = 1$

$\tan x = \pm\frac{1}{\sqrt{3}}$



$x \in \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

c) $\sin x + \cos x = 1$ $\quad (A+B)^2 = A^2 + B^2$ $\quad -10$



$\sin x = 1 - \cos x$

$\sin^2 x = 1 - 2\cos x + \cos^2 x$

$1 - \cos^2 x =$

$2\cos^2 x - 2\cos x = 0$

$2\cos x(\cos x - 1) = 0$

$\pm$ 2pts $\cos x = 0$ or $\cos x = 1$

$x \in \{0, \frac{\pi}{2}\}$

$x \in \{0, \frac{\pi}{2}, \cancel{\frac{3\pi}{2}}\}$

must check for extra roots,

$\sin x + \cos x = 1$

0: $0 + 1 = 1$, $1 = 1$

$\frac{\pi}{2}$: $1 + 0 = 1$, $1 = 1$

$\frac{3\pi}{2}$: $0 - 1 = 1$, $-1 = 1$ Reject

IV

$\cot^{-1}(x) = \begin{cases} \tan^{-1}\frac{1}{x} + \pi & \text{for } x < 0 \\ \tan\frac{1}{x} & \text{for } x > 0 \end{cases}$

a) let $\cot^{-1} x = b$ $\quad x > 0 \quad 0 < b < \frac{\pi}{2}$

$\cot(\cot^{-1} x) = \cot b$

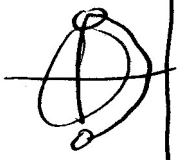
$x = \cot b$

$\frac{1}{x} = \tan b$

$\tan^{-1}\frac{1}{x} = \tan^{-1}(\tan b)$

$\tan^{-1}\frac{1}{x} = b$

$= \cot^{-1} x$



Extra.

$\sin 2x = \frac{1}{7}$ $\quad \sin^4 x + \cos^4 x = ?$

$2\sin x\cos x = \frac{1}{7}$

$4\sin^2 x\cos^2 x = \frac{1}{49}$

$2\sin^2 x\cos^2 x = \frac{1}{98}$

Now $(\sin^2 x + \cos^2 x) = 1$ $\quad$ Square both sides

$\sin^4 x + 2\sin^2 x\cos^2 x + \cos^4 x = 1$

$\sin^4 x + \frac{1}{98} + \cos^4 x = 1$

$\sin^4 x + \cos^4 x = \frac{97}{98}$

b) let $\cot^{-1} x = b$ $\quad x < 0 \quad \frac{\pi}{2} < x < \pi$

$\cot(\cot^{-1} x) = \cot b$

$x = \cot b$

$\frac{1}{x} = \tan b$

$\tan^{-1}\frac{1}{x} = \tan^{-1}(\tan b)$

but $\tan b = \tan(b+\pi) = \tan(b-\pi)$

$\frac{\pi}{2}, \pi \quad \frac{3\pi}{2}, 2\pi \quad -\frac{\pi}{2}, 0$

$\tan^{-1}(\frac{1}{x}) = \tan^{-1}(\tan(b-\pi))$

$= b - \pi$

$= \cot^{-1} x - \pi$ $\quad$ 4pts

$\tan^{-1}(\frac{1}{x}) + \pi = \cot^{-1} x$

Seen it but... $\quad -15$

11/22/02

	2	3	T
100	0	0	
90	2	1	
80	3	1	
70	6	3	
60	4	5	
59	10	17	
	1605	1459	24

99

$\frac{1605}{24} = 66.9$ $\frac{1459}{24} = 56.1$ $\frac{3064}{50}$

61.3

One Typo. two III.

Most students used copies of ~~other~~ previous students work.
They paid the price —
as predicted.
And as I warned them to be careful.
But what does the old boy know?

Honors Trigonometry

Name Trig Ch4

Per _____ Date ~~2002~~

Copy original problem.

Convince *me* that **you** understand the concept!

No Calculators!

Chapter 4 Exam

I State the Domain and Range, show a sketch of the function and show the "memory device" for each of the following: (20 pts tot)

a) $a(x) = \cos^{-1} x$ b) $b(x) = \sin^{-1} x$ c) $c(x) = \tan^{-1} x$

d) $d(x) = \sec^{-1} x$ e) $e(x) = \csc^{-1} x$ f) $f(x) = \cot^{-1} x$

II Given the constant k such that $0 \le k \le \pi$ and $\cos^{-1} x + \sin^{-1} x = k$.

Determine the exact value of k. (15 pts)

III Solve $\forall\ x\ \varepsilon\ [0, 2\pi)$ (i.e. $0 \le x < 2\pi$).
Use proper set notation and include "circle solutions" where appropriate. (15 pts ea.)

a) $\cos 2x + 2\sin^2 \frac{x}{2} = 1$ #51

b) $\tan 2x \tan x = 1$ #49

c) $\sin x + \cos x - 1 = 0$ #37

III Given: $\cot^{-1} x = \begin{cases} \tan^{-1} \frac{1}{x} + \pi & for\ x < 0 \\ \tan^{-1} \frac{1}{x} & for\ x > 0 \end{cases}$

For the following two parts, be *very* clear in your explanations.

a) Prove the $x > 0$ case. (15 pts)

b) Prove the $x < 0$ case. (5 pts)

Extra Credit ---------- 5 pts ----------

Given $\sin 2x = \frac{1}{7}$, find the exact value of $\sin^4 x + \cos^4 x$.

'02

Honors Trigonometry

				EXAM CHAPTER 3 No Homework *again!* ... and to think we get honors credit ... 11/1
Read ONLY: Pgs 109-114 and pgs 122 - 125 • • • • • • • • • • • • • • • • Do section I on back of HW sked # 1 11/4	# 2 Section II on back of sked #2 11/5	# 3 Prove: $\cos\left(\sin^{-1} u\right) = \sqrt{1-u^2}$ # 4 Pg 126 1 – 7, 19 – 24 11/6	# 5 Pg 121 7 – 21 Section V on back of sked 11/ 7	# 6 Section III back of sked # 7 R/C pg 137 11/8
School Holiday *Start work on # 8 Section IV on back of sked.* 11/11	Finish/ReDo Section IV on back of sked Quest for Quiz Points??? **(there are 5 other...)** 11/12	***"FREEDOM RINGS"*** ***It almost makes you want to march!*** Value: **10 assignments!!!** 11/13	***Have at least 1 - 20 done by the end of the day.*** 11/ 14	Continue having fun in Freedomland. Une autre la quizzette?? 11/ 15
When was the last time you looked at the "exam" door? ***You should have completed 1 - 60 by the beginning of the period tomorrow!*** 11/18	" 11/19		***Review*** ***Be prepared to put up last year's exam. Come to class prepared!*** 11/21	**Exam Ch 4** 11/22

Note: $\rightarrow \tan^{-1} x \neq \dfrac{\sin^{-1} x}{\cos^{-1} x}$

I

Find the value of each of the following:

a) $\cos^{-1}\frac{\sqrt{3}}{2}$ b) $\sin^{-1}\frac{1}{\sqrt{2}}$ c) $\sin^{-1}\frac{1}{2}$ d) $\cos^{-1}\left(-\frac{1}{2}\right)$ e) $\cos\left(\cos^{-1}\frac{1}{3}\right)$ f) $\sin\left(\sin^{-1}\frac{1}{5}\right)$

g) $\cos\left(\sin^{-1}\left(-\frac{4}{5}\right)\right)$ h) $\sin\left(\cos^{-1}\left(-\frac{5}{13}\right)\right)$ i) $\arcsin(\sin 3)$ j) $\arccos(\cos 5)$

II

Determine ALL values of u such that: a) $\left(\sqrt{u^2}\right) = u$ b) $\left(\sqrt{u}\right)^2 = u$

c) $\cos\left(\cos^{-1} u\right) = u$ d) $\cos^{-1}(\cos u) = u$ e) $\sin\left(\sin^{-1} u\right) = u$ f) $\sin^{-1}(\sin u) = u$

III

a) Pg 121 problem 22 b) When is this true? $\csc x = \dfrac{1}{\sin x}$ c) Determine $\sin^{-1} x = \csc^{-1}$?

d) complete: $\cos\left(\tan^{-1} u\right) = \cdots$ e) Simplify: $\sin\left(\sin^{-1}\frac{5}{13} + \tan^{-1}\frac{1}{2}\right)$ $\dfrac{22\sqrt{5}}{65}$

f) Simplify: $\cos\left(\tan^{-1}\frac{12}{13} + \sin^{-1}\frac{1}{2}\right)$ $\dfrac{13\sqrt{939} - 12\sqrt{313}}{626}$

IV

a) Do page 126 problems 25, 26, and 27

b) verify : (you may work with *both* sides.) : (1) $\cot^{-1} x = \tan^{-1}\frac{1}{x}$; $x > 0$ (2) $\tan^{-1} x = \frac{\pi}{2} - \cot^{-1} x$; $x > 0$

c) True or false? : $\cot^{-1} x = \tan^{-1}\frac{1}{x} + \pi$; $x < 0$ (Hint: graph $a(x) = \cot^{-1} x$ and $b(x) = \tan^{-1} x$ on the same axis.) Explain.

V

Determine ALL values of u such that:

a) $\tan\left(\tan^{-1} u\right) = u$ b) $\tan^{-1}(\tan u) = u$ c) $\csc^{-1}(\csc u) = u$ d) $\csc\left(\csc^{-1} u\right) = u$

e) $\sec\left(\sec^{-1} u\right) = u$ f) $\sec^{-1}(\sec u) = u$ g) $\cot\left(\cot^{-1} u\right) = u$ h) $\cot^{-1}(\cot u) = u$

I

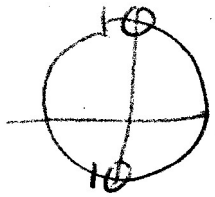
A) $\cos x - 2\sin^2 x + 2 = 0$

$\cos x - 2(1-\cos^2 x) + 2 = 0$

$2\cos^2 x + \cos x = 0$

$\cos x(2\cos x + 1) = 0$

$\cos x = -\frac{1}{2}$



10

$x \in \{\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}\}$

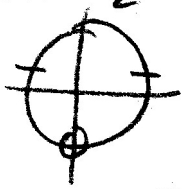
B) $2 + 2\sin x = 4\cos^2 x$

$1 + \sin x = 2 - 2\sin^2 x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ $\sin x = -1$



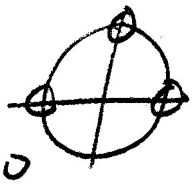
10

$x \in \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\}$

C) $\sin^2 x = \sin x$

$\sin x(\sin x - 1) = 0$

$\sin x = 0$ $\sin x = 1$



10

$x \in \{0, \frac{\pi}{2}, \pi\}$

D) $\cot^2 x - \cos^2 x = \cot^2 x \cos^2 x$

$\cot^2 x - \cot^2 x \cos^2 x - \cos^2 x = 0$

$\cot^2 x(1 - \cos^2 x) - \cos^2 x = 0$

$\cot^2 x \cdot \sin^2 x - \cos^2 x = 0$

$\cos^2 x - \cos^2 x = 0$

$0 = 0$

identity!

10

$x \in (0, 2\pi), x \neq \pi$

multiply by tans to get $1 - \sin^2 = \cos^2 x$

no. create list of $\frac{\pi}{2}, \frac{3\pi}{2}$ numerically

E) $\sec^2 2x + \tan 2x = 1$

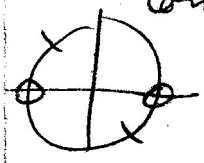
$s^2 + c^2 = 1$

$t^2 + 1 = \sec^2$

$\tan^2 2x + \tan 2x = 0$

$\tan 2x(\tan 2x + 1) = 0$

$\tan 2x = -1$



10

$x \in \{0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}, 2\pi, \frac{11\pi}{4}, 3\pi, \frac{15\pi}{4}\}$

$x \in \{0, \frac{3\pi}{8}, \frac{\pi}{2}, \frac{7\pi}{8}, \pi, \frac{11\pi}{8}, \frac{3\pi}{2}, \frac{15\pi}{8}\}$

2x around

-4

solve for x

-4

get text but ignore rest. -2

-3

F) $\cot x \tan 2x = 3$

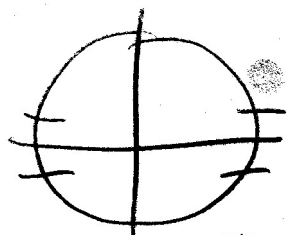
$\cot x \frac{2\tan x}{1 - \tan^2 x} = 3$

$2 = 3 - 3\tan^2 x$

$3\tan^2 x = 1$

$\tan x = \pm\frac{1}{\sqrt{3}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

I -3



10

$x \in \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

II

$A(x) = \sin^{-1} x$

D_A $|x| \leq 1$

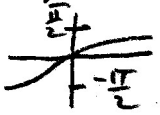
R_A $|A(x)| \leq \frac{\pi}{2}$

$B(x) = \cos^{-1} x$

D_B $|x| \leq 1$

R_B $0 \leq B(x) \leq \pi$

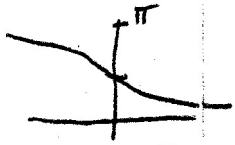
$C(x) = \tan^{-1} x$



D_C $x \in \mathbb{R}$

R_C $|C(x)| < \frac{\pi}{2}$

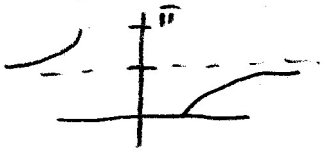
$D(x) = \cot^{-1} x$



D_D $x \in \mathbb{R}$

R_D $0 < D(x) < \pi$

$E(x) = \sec^{-1} x$



D_E $|x| \geq 1$

R_E $0 \leq E(x) \leq \pi$, $E(x) \neq \frac{\pi}{2}$

Bonus

III 25 total

A $\cos(\sin^{-1} u) = \sqrt{1-u^2}$

Let $\sin^{-1} u = y$ $\quad |y| \le \frac{\pi}{2}$, $|u| \le 1$

$\sin y = u$

$\sin^2 y = u^2$

$1 - \cos^2 y = u^2$

$\cos^2 y = 1 - u^2$

$\cos y = \pm\sqrt{1-u^2}$

$\cos(\sin^{-1} u) = \sqrt{1-u^2}$ QED

Since y in 1st or 4th Quad, cos of value in those Quads is > 0, NO $\pm$

B) $\csc^{-1}(u) = \sin^{-1}\left(\frac{1}{u}\right)$

let $y = \csc^{-1} u$ $\quad |u| \ge 1$, $|y| \le \frac{\pi}{2}$, $y \ne 0$

$\csc y = u$

$\frac{1}{\sin y} = u$

$\sin y = \frac{1}{u}$ $\quad$ since $|u| > 1$, $u \ne 0$

$y = \sin^{-1}\left(\frac{1}{u}\right)$

$\csc^{-1} u = \sin^{-1}\left(\frac{1}{u}\right)$

QED

C) $\csc(\cot^{-1} x) = A$

Let $y = \cot^{-1} x$ $\quad 0 < y < \pi$ $\quad x \in \mathbb{R}$

$\cot y = x$

$\cos y = x \sin y$

$\cos^2 y = x^2 \sin^2 y$

$1 - \sin^2 y = x^2 \sin^2 y$

$\sin^2 y (1 + x^2) = 1$

$\sin^2 y = \frac{1}{1+x^2}$

$\sin y = \pm \frac{1}{\sqrt{1+x^2}}$

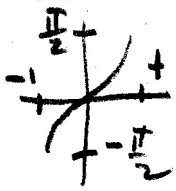
$\sin y = \frac{1}{\sqrt{1+x^2}}$ $\quad$ Since $y \in$ 1st or 2nd Quad & $\sin > 0$ there

$\csc y = \sqrt{1+x^2}$

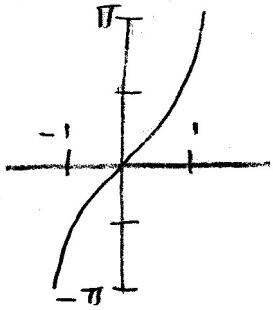
$\csc(\cot^{-1} x) = \sqrt{1+x^2}$

EXTRA.

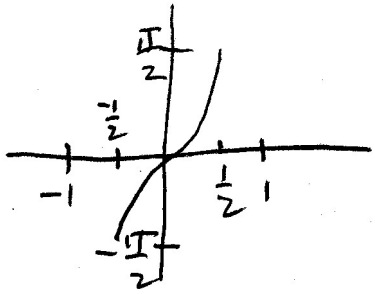
$\sin^{-1}$



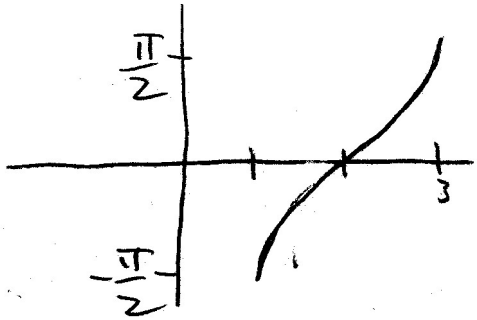
$2\sin^{-1} x$



$\sin^{-1} 2x$



$\sin^{-1}(x-2)$



100	1	1	2
90	7	5	12
80	9	4	13
70	6	3	9
60	4	6	10
59	5	2	7
	2444	1625	
	32	21	53
	76.375	77.38	76.8

Excellent exam
make only those
changes shown
other side.

I actually
expected grades to
be higher but
this is certainly better
than last year.

1990

100	1	0	1
90	5	7	12
80	9	8	17
70	6	12	18
60	1	0	1
59	1	1	2
			81.6

102

55

1899 ~~1609~~ / 23

2261 / 28

~~78.3~~ 82.6

80.75

10

55

1987

2.5 T

100	0	1	1
90	3	4	7
80	3	3	6
70	7	3	10
60	3	6	9
59	5	3	8
	73.2	74.7	73.9

many "copied" solutions on "freedom" at cost them when "faced for 1st time with some of these. they didn't do well at all on the proofs.

Honors Trigonometry Name Trig Ch4 1990

Copy the original problem.
Convince *me* that **you** understand the concept. Per _____ Date _____

CHAPTER #4

I

Solve $\forall x \quad 0 \le x < 2\pi$ (10 pts ea)

A) $\cos x - 2\sin^2 x + 2 = 0$

B) $2 + 2\sin x = 4\cos^2 x$

C) $\sin^2 x = \sin x$

D) $\cot^2 x - \cos^2 x = \cot^2 x \cos^2 x$

E) $\sec^2 2x + \tan 2x = 1$

F) $\cot x \tan 2x = 3$

II

State the Domain and Range AND sketch the function (3 pts ea)

A) $A(x) = \sin^{-1} x$ B) $B(x) = \cos^{-1} x$ C) $C(x) = \tan^{-1} x$ D) $D(x) = \cot^{-1} x$ E) $E(x) = \sec^{-1} x$

III

State ALL restrictions on "u" AND on any other variables you may use.
Justify *any* questionable (by me) steps. (25 pts tot)

A) PROVE: $\cos(\sin^{-1} u) = \sqrt{1 - u^2}$

B) PROVE: $\csc^{-1} u = \sin^{-1}\left(\frac{1}{u}\right)$

C) Determine what: $\csc(\cot^{-1} u)$ is equal to.
(hint: It might be something like the answer to part A.)

EXTRA CREDIT 5 pts

Using the standard function $F(x) = \sin^{-1} x$ as a starting point,

sketch each of the following on separate axes.

$A(x) = 2\sin^{-1} x$ $B(x) = \sin^{-1} 2x$ $C(x) = \sin^{-1}(x - 2)$

I 10 pts ea

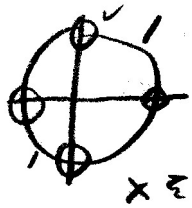
A) $2\cos^2 x \sin^2 x = \cos x \sin x$

$\cos x \sin x (2\sin x \cos x - 1) = 0$

$\cos x = 0$ $\quad$ $\sin 2x = 1$ $\quad$ not using $2x$ $\quad -2$

$\sin x = 0$ $\quad$ $2x \in \{\frac{\pi}{2}, \frac{5\pi}{2}\}$

$x \in \{\frac{\pi}{4}, \frac{5\pi}{4}\}$



$x \in \{0, \frac{\pi}{4}, \frac{\pi}{2}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}\}$

B) $\sin x + \sec^2 x = 1 + \sin^2 x + \tan^2 x$

$s^2 + t^2 = 1$

$t^2 + 1 = \sec^2 x$

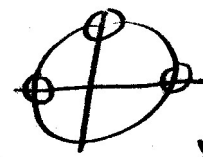
$\sin x + \tan^2 x + 1 = 1 + \tan^2 x + \sin^2 x$

$\sin^2 x - \sin x = 0$

$\sin x(\sin x - 1) = 0$ $\quad$ 2 pts

Reject $x = \frac{\pi}{2}$

$\tan x$ not defined

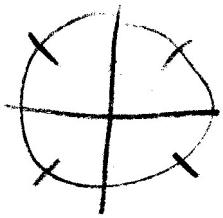


$x \in \{0, \pi\}$

C) $\tan^2 x = 1$ $\quad$ 3

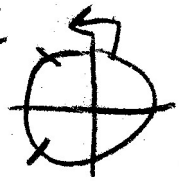
$\tan x = \pm 1$

$x \in \{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$



D) $\sin^2(x + \frac{\pi}{6}) - \cos^2(x + \frac{\pi}{6}) = \frac{\sqrt{2}}{2}$

$\cos(2x + \frac{\pi}{3}) = -\frac{\sqrt{2}}{2}$



$0 \le x < 2\pi$

$\frac{\pi}{3} \le 2x + \frac{\pi}{3} < 4\pi + \frac{2\pi}{3}$

$2x + \frac{\pi}{3} \in \{\frac{3\pi}{4}, \frac{5\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}\}$

$x \in \{\frac{5\pi}{24}, \frac{11\pi}{24}, \frac{29\pi}{24}, \frac{35\pi}{24}\}$

E) $3\csc^2 x + \sqrt{3}\csc x = 0$

$\csc x(3\csc x + \sqrt{3}) = 0$

$\csc x = 0$ $\quad$ $\csc x = -\frac{\sqrt{3}}{3} = -\frac{1}{\sqrt{3}}$

Reject

Range violation

$\sin x = -\sqrt{3}$

This too, Range violation

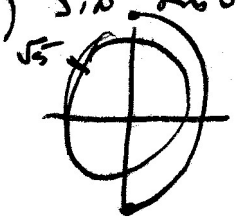
II

5 pts ea

A) $\text{Arcsin}\,\frac{1}{\sqrt{2}}$

$\frac{\pi}{4}$

B) $\sin^{-1}\sin\sqrt{5} =$

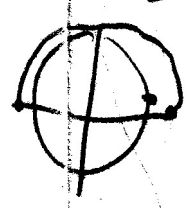


$\pi - \sqrt{5}$

2

C) $\sec^{-1}\frac{2}{\sqrt{3}} =$

$\cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$



D) $\cos(\text{arctan } 1)$

$\cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$.

III 10 pts ea

A) $\sin(\cos^{-1}u) = \sqrt{1-u^2}$

let $\cos^{-1}u = b$ $\quad |u| \le 1 \quad 0 \le b \le \pi$

$\cos b = u$

$\cos^2 b = u^2$

$1 - \sin^2 b = u^2$

$\sin^2 b = 1 - u^2$

$\sin b = \pm\sqrt{1-u^2}$

Reject − sign. $b \in 1^{st}$ & 2nd Quad & $\sin > 0$ in this interval

$\sin(\cos^{-1}u) = \sqrt{1-u^2}$ QED

c) $\cos^{-1}u = \sec^{-1}\left(\frac{1}{u}\right)$

let $\cos^{-1}u = b$ $\quad |u| \le 1, u \ne 0$

$0 \le b \le \pi, b \ne \frac{\pi}{2}$

$\cos b = u$

$\frac{1}{\cos b} = \frac{1}{u}$

$\sec b = \frac{1}{u}$

$b = \sec^{-1}\left(\frac{1}{u}\right)$

$\cos^{-1}u = \sec^{-1}\left(\frac{1}{u}\right)$

QED

B) $\cos(\tan^{-1}u) = \frac{1}{\sqrt{1+u^2}}$

let $\tan^{-1}u = b$ $\quad u \in \mathbb{R}, |b| < \frac{\pi}{2}$

$\tan b = u$

$\sin^2 b = u^2\cos^2 b$

$1 - \cos^2 b = u^2\cos^2 b$

$\cos^2 b(1+u^2) = 1$

$\cos^2 b = \frac{1}{1+u^2}$ reject the − sign.

$\cos b = \frac{\pm 1}{\sqrt{1+u^2}}$ $\quad b \in 1^{st}$ & 4th Quad. $\cos > 0$

$\cos(\tan^{-1}u) = \frac{1}{\sqrt{1+u^2}}$ QED

EXTRA

$\cos^{-1}x + \sin^{-1}x = K$

$\cos(\cos^{-1}x + \sin^{-1}x) = \cos K$

$\cos\cos^{-1}x \cos\sin^{-1}x - \sin(\cos^{-1}x)\sin(\sin^{-1}x) =$

$x\sqrt{1-x^2} - x\sqrt{1-x^2} = \cos K$

$0 = \cos K$

$\therefore K = \frac{\pi}{2}$.

Notice $(0 \le \cos^{-1}x \le \pi) + \left(-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}\right)$

The least K could be is intuition

$0 \le K \le \frac{\pi}{2}$.

$\sin(\cos^{-1}x + \sin^{-1}x) = \sin K$

$\sin\cos^{-1}x \cos\sin^{-1}x + \sin\sin^{-1}x \cos\cos^{-1}x$

$\sqrt{1-x^2} \cdot \sqrt{1-x^2} + x \cdot x$

$1 - x^2 + x^2$

$1 = \sin K$

$\therefore K = \frac{\pi}{2}$

85

Excellent exam.
No changes necessary —
I guess more quizzes are needed.
maybe giving ANS to part of
freedom ditto would help.
Yes. I think that is a good
idea.
also lecture on using double
angles is vital.

Cos 2x = 1 ∴ 2 times around etc.

	2	5	I	
100	0	0	0	99
90	4	6	10	
80	2	3	5	
70	6	4	10	
60	6	6	12	
59	7	6	13	
	69.8	72.5	71.1	22

88

	2	5	T
100	0	1	1
90	4	4	8
80	5	2	7
70	6	4	10
60	3	2	5
59	8	3	11
	67.0	75.6	70.5

This is "about" the same as the other 2 times!
What is hard about this?

I D is lecture pblm.

I B } could be tricky.
E }

Most DID very poor on proofs.

I conclude: remove III A
Section III now 15 pts ea.

91

100	0	0
90	1	7
80	8	8
70	2	5
60	2	2
59	2	3

99

40

$\frac{1112}{15}$ $\frac{1983}{25}$ 77.38

74.13 79.32

77. 80.7

I guess this is as tough as it can be made. Both bottom scores are pretenders

83

	5	6	5
100	1	1	2
90	0	3	3
80	9	3	12
70	8	5	13
60	5	3	8
59	6	10	16
	29	25	

$\frac{2043}{29}$ $\frac{1643}{25}$ $= \frac{3686}{54}$

70.45 65.72 68.

Honors Trigonometry
Copy original problem.
Convince *me* that **you** understand the concept!

Name TRG Ch4

Per _____ Date 11/22/91

Chapter 4 Exam

I Solve $\forall\ x:\ 0 \le x < 2\pi$ (10 pts ea)

A) $2\cos^2 x\ \sin^2 x = \cos x\ \sin x$

B) $\sin x + \sec^2 x = 1 + \sin^2 x + \tan^2 x$

C) $\tan^2 x = 1$

D) $\sin^2\left(x + \frac{\pi}{6}\right) - \cos^2\left(x + \frac{\pi}{6}\right) = \frac{\sqrt{2}}{2}$

E) $3\csc^2 x + \sqrt{3}\csc x = 0$

II Find the value of the following: (5 pts ea)

A) $\arcsin\frac{1}{\sqrt{2}}$

B) $\sin^{-1}\left(\sin\sqrt{5}\right)$

C) $\sec^{-1}\frac{2}{\sqrt{3}}$

D) $\cos(\arctan 1)$

III State all restrictions on "u" and on any other variables you may use.
Justify *any* questionable steps. (15 pts ea)

A) Prove: $\sin\left(\cos^{-1} u\right) = \sqrt{1-u^2}$

B) Prove: $\cos\left(\tan^{-1} u\right) = \frac{1}{\sqrt{1+u^2}}$

Extra Credit ═══ **5 pts** ═══

IF $\cos^{-1} x + \sin^{-1} x$ is equal to a particular constant

THEN determine the value of that constant

ELSE find an expression in terms of x to which $\cos^{-1} x + \sin^{-1} x$ is equal.

I

A) $\tan 2x = \frac{3}{\cot x}$

$\frac{2\tan x}{1-\tan^2 x} = \frac{3}{\cot x}$

$2 = 3 - 3\tan^2 x$

$3\tan^2 x = 1$

$\tan x = \pm\frac{1}{\sqrt{3}} = \frac{1}{2}$, $\frac{\sqrt{3}}{2}$

$x \approx \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

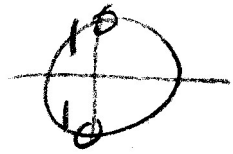
B) $\cos x - 2\sin^2 x + 2 = 0$

$\cos x - 2(1-\cos^2 x) + 2 = 0$

$\cos x - \cancel{2} + 2\cos^2 x + \cancel{2} = 0$

$\cos x(1 + 2\cos x) = 0$

$\cos x = -\frac{1}{2}$



$x \approx \{\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}\}$

-3 Don't see $\tan\frac{\pi}{2}$ as problem

-1 See it but don't Handle it

C) $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$

Don't use "add" formula

-5

$\cos(\qquad) = \cos\frac{\pi}{2}$

$\cos\tan^{-1}(1+x)\cos(\tan^{-1}1-x) - \sin(\tan^{-1}(1+x)\sin(\tan^{-1}(1-x) = 0$

$\frac{1}{\sqrt{1+(1+x)^2}} \cdot \frac{1}{\sqrt{1+(1-x)^2}} - \frac{1+x}{\sqrt{1+(1+x)^2}} \cdot \frac{1-x}{\sqrt{1+(1-x)^2}} = 0$

$1 - (1-x^2) = 0$

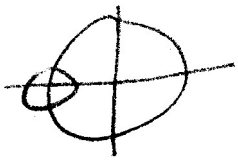
$1 - 1 + x^2 = 0$

$x = 0$

E) $\sin 2x \sin x - \cos 2x \cos x = 1$

$\cos(2x + x) = -1$

$\cos 3x = -1$



$3x \approx \{\pi, 3\pi, 5\pi\}$

$x = \{\frac{\pi}{3}, \pi, \frac{5\pi}{3}\}$,

D) $\sec^2 2x + \tan 2x = 1$

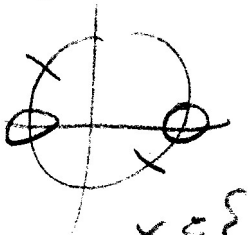
$s^2 + c^2 = 1$

$c^2 + 1 = \sec^2$

$\tan^2 2x + 1 + \tan 2x = 1$

$\tan 2x(\tan 2x + 1) = 0$

$\tan 2x = -1$



$2x \approx \{0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}, 2\pi, \frac{11\pi}{4}, 3\pi, \frac{15\pi}{4}\}$, not $2x$ around

-5

"NOT = 0" -5

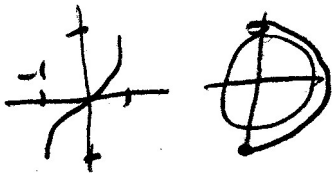
$x \approx \{0, \frac{3\pi}{8}, \frac{\pi}{2}, \frac{7\pi}{8}, \pi, \frac{11\pi}{8}, \frac{3\pi}{2}, \frac{15\pi}{8}\}$

II

A) $A(x) = \sin^{-1} x$

D_A $|x| \le 1$

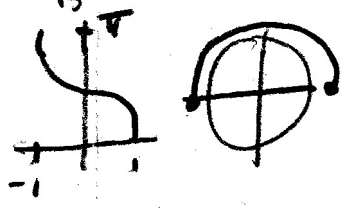
R_A $|A(x)| \le \frac{\pi}{2}$



B) $B(x) = \cos^{-1} x$

D_B $|x| \le 1$

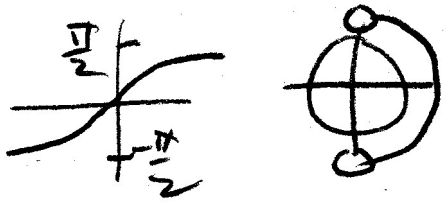
R_B $0 \le B(x) \le \pi$



C) $C(x) = \tan^{-1} x$

D_C $x \in \mathbb{R}$

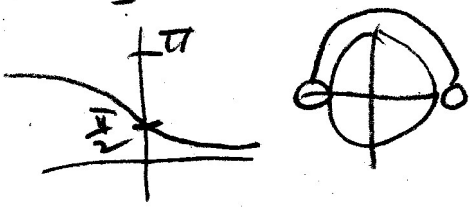
R_C $|C(x)| < \frac{\pi}{2}$



D) $D(x) = \cot^{-1} x$

D_D $x \in \mathbb{R}$

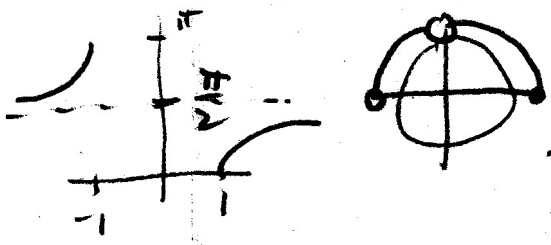
R_D $0 < D(x) < \pi$



E) $E(x) = \sec^{-1} x$

D_E $|x| \ge 1$

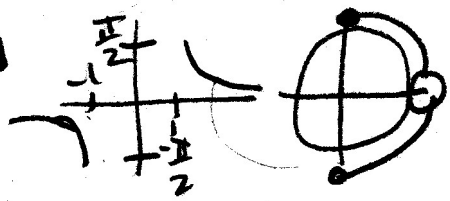
R_E $0 \le E(x) \le \pi$; $E(x) \ne \frac{\pi}{2}$



F) $f(x) = \csc^{-1} x$

D_F $|x| \ge 1$

R_f $|f(x)| \le \frac{\pi}{2}$; $f(x) \ne 0$



III $\sec \tan^{-1} x = \sqrt{x^2+1}$

Let $\tan^{-1} x = b$ $x \in \mathbb{R}$, $|b| < \frac{\pi}{2}$

$\tan(\tan^{-1} x) = \tan b$

$x = \tan b$

$x^2 = \tan^2 b$

$x^2+1 = \tan^2 b + 1$

$x^2+1 = \sec^2 b$

$\pm\sqrt{x^2+1} = \sec b$

$b \in 1^{st}, 4^{th}$ Quads, 2 pts

Sec of $b > 0$

$\sqrt{x^2+1} = \sec b$

$\sqrt{x^2+1} = \sec(\tan^{-1} x)$

$s^2 + c^2 = 1$

$t^2 + 1 = \sec^2$

Extra,

$(3a+2b)x + (5a+3b)$

$ax^4 + (a+b)x^3 + (2a+b)x^2$

$x^2 - x - 1 \overline{)\, ax^6 + bx^5 \quad +1}$

$ax^6 - ax^5 - ax^4$

$(a+b)x^5 + ax^4$

$(a+b)x - (a+b)x^4 - (a+b)x^3$

$(2a+b)x^4 + (a+b)x^3$

$(2a+b)x^4 + (2a+b)x^3 - 2a+bx^2$

$(3a+2b)x^3 + (2a+b)x^2$

$(3a+2b)x^3 + (3a+2b)x - 3a+2bx$

$(5a+3b)x^2 + (3a+2b)x$

$(5a+3b)x^2 - (5a+3b)x - 5a+3b$

$+1$

$(8a+5b)x + 5a+3b+1 = 0$

$8a+5b=0$

$-5a+3b=-1$

$24a+15b=0$

$-25a-15b=+5$

$-a = -5$

$a=-5$

$-40+5b=0$

$b=8$

$a+b=3$

Remainder had to be zero.

	2	5	7
100	0	0	
90	6	6	
80	5	7	
70	0	3	
60	4	3	
55	5	4	

98

32

$\frac{1468}{20}$	$\frac{1764}{23}$	$\frac{3232}{43}$
73.4	76.7	75.2

The extra credit was too hard. Not that any of this group got to it. They were too busy mining my problems — from the HW — to do any extra — Definitely!

oh — those who copied freedom Rings did fail exam.

Do Not add any problems to this length!!!

Honors Trigonometry Name Trig ch 4 9J

Copy original problem. Per _____ Date _____

Convince *me* that **you** understand the concept!

Chapter 4 Exam

I

Solve $\forall\ x\ \varepsilon\ [0,2\pi)$ (i.e. $0 \le x < 2\pi$).
Include "circle solutions" as well as proper set notation. (15 pts ea.)

a) $\tan 2x = \dfrac{3}{\cot x}$

b) $\cos x - 2\sin^2 x + 2 = 0$

c) $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \dfrac{\pi}{2}$

d) $\sec^2 2x + \tan 2x = 1$

e) $\sin 2x \sin x - \cos 2x \cos x = 1$

II

For each of the following functions, state the domain and range, draw the graph of the function on a properly labeled axis and show the "memory device" we use for the function. (tot 15 pts)

a) $A(x) = \sin^{-1} x$

b) $B(x) = \cos^{-1} x$

c) $C(x) = \tan^{-1} x$

d) $D(x) = \cot^{-1} x$

e) $E(x) = \sec^{-1} x$

f) $F(x) = \csc^{-1} x$

III

Prove: $\sec\left(\tan^{-1} x\right) = \sqrt{x^2+1}$. Be sure to describe restrictions on any variables, as usual. (10 pts)

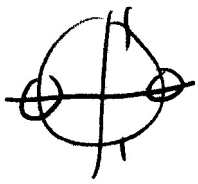
Extra Credit ----------5 pts----------

If $x^2 - x - 1$ divides $ax^6 + bx^5 + 1$ evenly, find the sum $a + b$.

I a) $\sin 2x = \sin x$

$2\sin x\cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$



$x = \{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}\}$

b) $\sin x - \sqrt{3}\cos x = 1$

$3\cos^2 x = 1 - 2\sin x + \sin^2 x$

$3 - 3\sin^2 x =$

$4\sin^2 x - 2\sin x - 2 = 0$

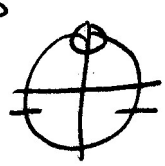
$2\cdot(2\sin^2 x - \sin - 1) = 0$

$2(2\sin x + 1)(\sin x - 1) = 0$

$2\sin x = -1$ $\quad \sin x = 1$

$\sin x = -\frac{1}{2}$

$(A+B)^2 = A^2 + B^2$ yields -3 pts



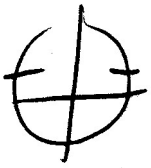
$x = \{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

must show check $\frac{11\pi}{6}$ Doesn't work

$x = \{\frac{\pi}{2}, \frac{7\pi}{6}\}$

c) $2\sin 3x - 1 = 0$

$\sin 3x = \frac{1}{2}$ $\quad$ use sin add & get lost $\to$



$3x = \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}\}$

$x = \{\frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}\}$

d) $\cos^2(x + \frac{\pi}{3}) - \sin^2(x + \frac{\pi}{3}) = \frac{1}{2}$

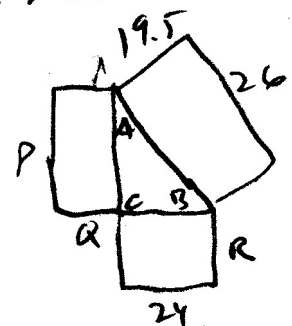
$\cos 2(x + \frac{\pi}{3}) = \frac{1}{2}$

$2(x + \frac{\pi}{3}) = \{\frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}\}$

$x + \frac{\pi}{3} = \{\frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}\}$

$x = \{\frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\}$

Extra credit



$26^2 = 24^2 + P^2$

$676 - 576 = P^2$

$P = 10$

$\frac{19.5}{26} = .75$ common ratio?

A

$507 = 10Q + 24R$

$10\cdot(7.5) + 24\cdot 18$

$\underbrace{75 + 432}_{507}$

$P = 10 \quad Q = 7.5 \quad R = 18$

I E) $\tan\left(2x - \frac{3\pi}{2}\right) = \cot x$ used tan add

$$\frac{\sin 2x \underbrace{\cos\frac{3\pi}{2}}_{0} - \sin\frac{3\pi}{2}\cos 2x}{\cos 2x \underbrace{\cos\frac{3\pi}{2}}_{0} + \sin 2x \sin\frac{3\pi}{2}} = \frac{\cos x}{\sin x}$$

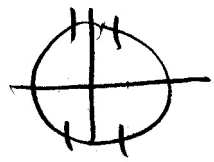
-5

$$\frac{\cos 2x}{-\sin 2x} = \frac{\cos x}{\sin x}$$

$$-2\sin x\cos x\left(\frac{\cos^2 x - \sin^2 x}{-2\sin x\cos x} = \frac{\cos x}{\sin x}\right)$$

$$\cos^2 x - (1-\cos^2 x) = -2\cos^2 x$$

$$2\cos^2 x - 1 =$$



$$4\cos^2 x = 1$$

$$\cos x = \pm\frac{1}{2}$$

$$x \in \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$$

II

$$\cos^{-1}x + \sin^{-1}x = K$$

$$\cos\cos^{-1}x \cos\sin^{-1}x - \sin(\cos^{-1}x)\sin(\sin^{-1}x) = \cos K$$

$$x\sqrt{1-x^2} - x\sqrt{1-x^2} = \cos K$$

$$0 = \cos K$$

$$K = \frac{\pi}{2}.$$

II $\cos^{-1}x + \cos^{-1}2x = \frac{\pi}{3}$

$$\cos^{-1}2x = \frac{\pi}{3} - \cos^{-1}x$$

NO cos Add

-8

$$2x = \cos\left(\frac{\pi}{3}\right)\cos(\cos^{-1}x) + \sin\frac{\pi}{3}\sin(\cos^{-1}x)$$

$$2x = \frac{1}{2}x + \frac{\sqrt{3}}{2}\sqrt{1-x^2}$$

$$4x = x + \sqrt{3}\sqrt{1-x^2}$$

$$3x = \sqrt{3}\sqrt{1-x^2}$$

$$\frac{9x^2}{3} = 1 - x^2$$

$$4x^2 = 1$$

$$x = \pm\frac{1}{2}$$

$$\cos^{-1}\left(\frac{1}{2}\right) + \cos^{-1}(1) = \frac{\pi}{3}$$

$$\frac{\pi}{3} + 0 = \frac{\pi}{3} \checkmark$$

$$\cos^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}(-1) = \frac{\pi}{3}$$

$$\frac{\pi}{3} + \pi = \frac{\pi}{3}$$ NO Reject

$$x = \frac{1}{2} \text{ only.}$$

$$\cos(\cos^{-1}x + \cos^{-1}2x) = \cos\frac{\pi}{3}$$

$$(x)(2x) - \sqrt{1-x^2}\sqrt{1-4x^2} = \frac{1}{2}$$

$$2x^2$$

$$4x^2 - 1 = \sqrt{1-x^2}\sqrt{1-4x^2}$$

$$16x^4 - 8x^2 + 1 = (1-x^2)(1-4x^2)$$

$$= 1 - 5x^2 + 4x^4$$

$$12x^4 - 3x^2 = 0$$

$$3x^2(4x^2 - 1) = 0$$

$$x^2 = \frac{1}{4}$$

$$x = \pm\frac{1}{2}$$

$$x = 0$$

$$\cos(\cos^{-1}x = K - \sin^{-1}x)$$

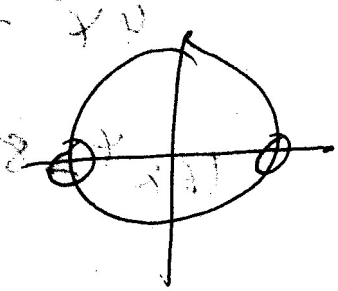
$$x = \cos K \cos\sin^{-1}x + \sin K \sin(\sin^{-1}x)$$

$$\text{ugh} = \cos K\sqrt{1-x^2} + x\sin K$$

$$\frac{\cos 2x}{-\sin 2x} - \frac{\cos x}{\sin x} = 0$$

$$\underbrace{\sin x \cos 2x + \sin 2x \cos x}_{\sin 3x} = 0$$

$$\sin 3x = 0$$



$$3x \in \{0 \quad \pi \quad 2\pi \quad 3\pi \quad 4\pi \quad 5\pi$$

$$\cancel{0} \quad \frac{\pi}{3} \quad \frac{2\pi}{3} \quad \cancel{\pi} \quad \frac{4\pi}{3} \quad \frac{5\pi}{3}$$

$$\cot x = \frac{\cos x}{\sin x}$$

$x = \pi$

$\therefore$ $x = 0$ not good solutions.

Doesn't check.

	5	6	T	
100	0	0	0	97
90	5	2	7	
80	10	5	15	
70	3	8	11	
60	2	0	2	
59	8	11	19	
				18
	$\frac{2058}{28}$	$\frac{1708}{26}$	~~69.7~~	
	~~73.5~~ 75.6	65.7	70.7	

almost — The extra credit really should be replaced.

Great exam —

Students feared Proofs
and there were none. They are disappointed.
why? because "they couldn't do proofs"
was the ready excuse but was unneeded

Section I was lecture problems.
The main error throughout was
very poor algebra skills.
and sloppy trig work
Primarily sin add and cos add.

Honors Trigonometry Name TRIG C4 95

Copy original problem. Per _____ Date _____ 11/28

Convince *me* that **you** understand the concept!

Chapter 4 Exam

I Solve $\forall \quad 0 \le x < 2\pi$ **(15 pts ea)**

a) $\sin 2x = \sin x$

b) $\sin x - \sqrt{3}\cos x = 1$

c) $2\sin 3x - 1 = 0$

d) $\cos^2\left(x + \frac{\pi}{3}\right) - \sin^2\left(x + \frac{\pi}{3}\right) = \frac{1}{2}$

e) $\tan\left(2x - \frac{3\pi}{2}\right) = \cot x$

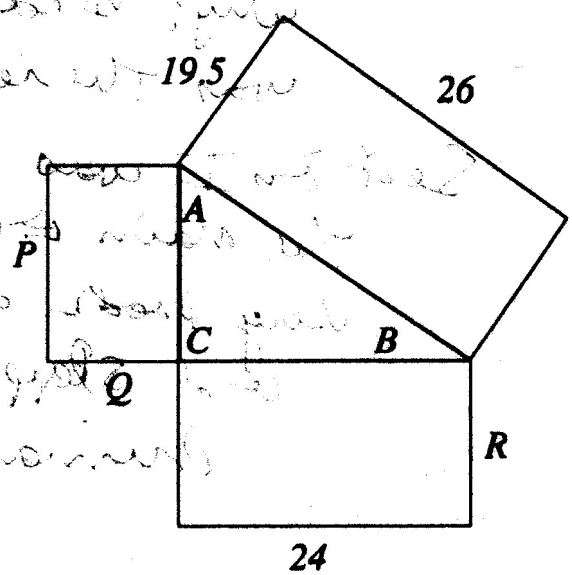
for final

II Solve for x: $\cos^{-1} x + \cos^{-1} 2x = \frac{\pi}{3}$ **(15 pts)**

III Given $0 \le k < \pi$. $\cos^{-1} x + \sin^{-1} x = k$. Solve for k. **(10 pts)**

Extra Credit ---------- **5 pts** ----------

On right triangle ABC, three rectangles have been drawn. (Alas, the drawing is not to scale.) Determine the values of P, Q, and R so that the sum of areas of the "leg rectangles" equals the area of the "hypotenuse rectangle."



I

a) $\cos x - 2\sin^2 x + 2 = 0$

$\cos x - 2(1-\cos^2 x) + 2 = 0$

$2\cos^2 x + \cos x = 0$

$\cos x(2\cos x + 1) = 0$

$\cos x = -\frac{1}{2}$

$x \in \{\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}\}$

b) $\sin x + \underline{\sec^2 x} = 1 + \sin^2 x + \underline{\tan^2 x}$

$s^2 + c^2 = 1$

$t^2 + 1 = s^2 c^2$ subtraction

$\sin x = \sin^2 x$

$\sin^2 x - \sin x = 0$

$\sin x(\sin x - 1) = 0$

$x \in \{0, \frac{\pi}{2}, \pi\}$

Rejected ↑ tangent undefined.

2pts

$x \in \{0, \pi\}$

c) $\sin^2 x - 2\sin x + 1 = 0$

$(\sin x - 1)^2 = 0$

$x \in \{\frac{\pi}{2}\}$

d) $2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = -\frac{1}{2}$ $\quad \sin x = 1$

$x \in \{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

e) $\sin 2x + \cos 2x - 1 = 0$

$(\cos 2x - 1) = -\sin 2x$

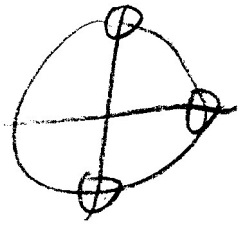
$\cos^2 2x - 2\cos 2x + 1 = \sin^2 2x$

$= 1 - \cos^2 2x$

must check for extraneous roots.

$2\cos^2 2x - 2\cos 2x = 0$

$2\cos 2x(\cos 2x - 1) = 0$



0	$\sin 0 + \cos 0 - 1 = 0$	$1 - 1 = 0$ OK
$\frac{\pi}{4}$	$1 + 0 - 1 = 0$	OK
$\frac{3\pi}{4}$	$-1 + 0 - 1 =$	NO
π	$0 + 1 - 1 = 0$	OK
$\frac{5\pi}{4}$	$-1 + 0 - 1 = 0$	OK
$\frac{7\pi}{4}$	$-1 + 0 - 1 = -2$	NO

$2x \in \{0, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}, \frac{7\pi}{2}\}$

$x \in \{0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}\}$

↑ NO ↑ NO

$x \in \{0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}\}$

II $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{12}{13} = \cos^{-1}\frac{-x}{65}$

$\cos\sin^{-1}\frac{3}{5}\cos\sin^{-1}\frac{12}{13} - \sin\sin^{-1}\frac{3}{5}\sin\sin^{-1}\frac{12}{13} = \cos(\cos^{-1}\frac{-x}{65})$

$\sqrt{1-\frac{9}{25}}\sqrt{1-\frac{144}{169}} - \frac{3}{5}\cdot\frac{12}{13} = \frac{-x}{65}$

$\frac{4}{5}\cdot\frac{5}{13} - \frac{36}{65}$

$\frac{-16}{65} = \frac{-x}{65}$ $\quad x = 16$

didn't take of cos inside so both sides wrong but you did. -6

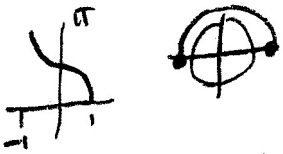
Distributive formula instead of sin/cos add -6

III

$A(x) = \cos^{-1}x$

D_A $-1 \le x \le 1$

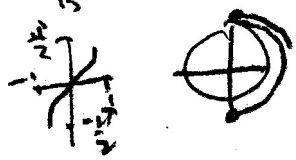
R_A $0 \le A(x) \le \pi$



$B(x) = \sin^{-1}x$

D_B $-1 \le x \le 1$

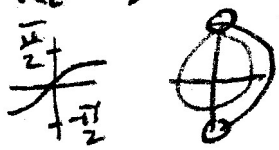
R_B $-\frac{\pi}{2} \le B(x) \le \frac{\pi}{2}$



$C(x) = \tan^{-1}x$

D_C $x \in \mathbb{R}$

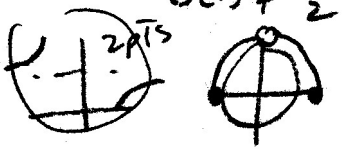
R_C $-\frac{\pi}{2} < C(x) < \frac{\pi}{2}$



$D(x) = \sec^{-1}x$

D_D $x \le -1$ or $x \ge 1$

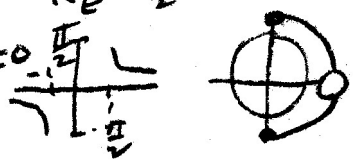
R_D $0 \le D(x) \le \pi$, $D(x) \ne \frac{\pi}{2}$



$E(x) = \csc^{-1}x$

D_E $x \le -1$ or $x \ge 1$

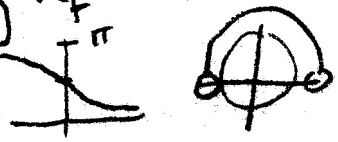
R_E $-\frac{\pi}{2} \le E(x) \le \frac{\pi}{2}$



$f(x) = \cot^{-1}x$

D_f $x \in \mathbb{R}$

R_f $0 < f(x) < \pi$



IV $\csc(\cot^{-1}u) = \sec(\tan^{-1}u)$ $\quad u > 0$

$\csc(\cot^{-1}u) = \frac{1}{\sin(\tan^{-1}\frac{1}{u})} = \frac{1}{\frac{\frac{1}{u}}{\sqrt{1+\frac{1}{u^2}}}}$

$\frac{\sqrt{1+\frac{1}{u^2}}}{\frac{1}{u}} = \frac{\sqrt{\frac{u^2+1}{u^2}}}{\frac{1}{u}} = \frac{u}{\sqrt{u^2+1}} =$

$\sec(\tan^{-1}u) = \frac{1}{\cos\tan^{-1}u} = \frac{1}{\frac{1}{\sqrt{1+u^2}}} = \sqrt{1+u^2}$

Full effort but not conclusive. -3

$\tan^{-1} = \frac{\sin^{-1}}{\cos^{-1}}$ $\quad -5$

$\frac{\sin x\cos x}{1-2\sin^2 x} = \frac{\sin x\cos x}{\cos^2 x - \sin^2 x}$

$= \frac{\sin x\cos x}{1-2\sin^2 x}$ $\quad +2$

Identity!

Denom set $= 0$.

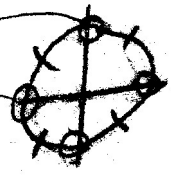
$(\cos x - \sin x)(\cos x + \sin x) = 0$

EXTRA

$\frac{\sin x\cos x}{1-2\sin^2 x} = \frac{1}{\cot x - \tan x}$

$= \frac{1}{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}}$

$= \frac{1}{\frac{\cos^2 x - \sin^2 x}{\sin x\cos x}}$



$+5$

$x \notin \{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$

$\{0, \frac{\pi}{2}, \frac{3\pi}{2}, \pi\}$

	5	6	T
100	0	1	1
90	5	5	10
80	7	4	11
70	5	6	11
60	4	2	6
59	0	1	1
	1700/21	1519/19	80.48

5th 81.0

6th 79.9

11/25/96

100	4
90	2
80	6
70	7
60	3
59	4
	77.8

102

43

ERROR on I B

I gave up 10 pts if student made absolutely any attempt.

Nobody did IB in the desired manner. Too hard?

No they Proved it rather than use the "tricks"

I_E is probably easier if you change to single angle then you won't have to check answers.

Honors Trigonometry Name Trig Ch 4

Copy original problem. Per _____ Date 11/29/94

Convince *me* that you understand the concept! 11/25/96

Chapter 4 Exam

I

Solve $\forall\ x\ \ 0 \le x < 2\pi$ **(10 pts ea)**

a) $\cos x - 2\sin^2 x + 2 = 0$

b) $\sin x + \sec^2 x = 1 + \sin^2 x + \tan^2 x$

c) $\sin^2 x - 2\sin x + 1 = 0$

d) $2\sin^2 x - \sin x - 1 = 0$

e) $\sin 2x + \cos 2x - 1 = 0$

II

Solve $\forall\ x\ \varepsilon\ \Re$ **(10 pts)**

$$\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{12}{13} = \cos^{-1}\frac{-x}{65}$$

III

State the Domain and Range, show a sketch of the function and show the "memory device" for each of the following: **(25 pts tot)**

a) $a(x) = \cos^{-1} x$

b) $b(x) = \sin^{-1} x$

c) $c(x) = \tan^{-1} x$

d) $d(x) = \sec^{-1} x$

e) $e(x) = \csc^{-1} x$

f) $f(x) = \cot^{-1} x$

IV

Given $u > 0$. Show that $\csc\left(\cot^{-1} u\right) = \sec\left(\tan^{-1} u\right)$.

Be very clear in your method. Explain what you are doing.
(HINT: You are *not* required to show a proof.) **(15 pts)**

Extra Credit ---------- **5 pts** ----------

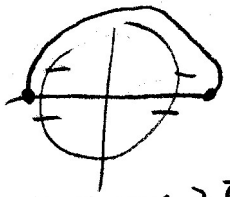
Solve $\forall\ x\ \ 0 \le x < 2\pi$: $\dfrac{\sin x \cos x}{1 - 2\sin^2 x} = \dfrac{1}{\cot x - \tan x}$

I

a) $3\cos\frac{x}{2} = \cot\frac{x}{2}$

$\cos^2\frac{x}{2} = \frac{1}{3}$

$\cos\frac{x}{2} = \pm\frac{1}{\sqrt{3}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$



$0 \le x < 2\pi$

$0 \le \frac{x}{2} < \pi$

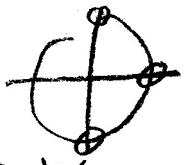
$\frac{x}{2} = \{\frac{\pi}{6}, \frac{5\pi}{6}\}$

$x = \{\frac{\pi}{3}, \frac{5\pi}{3}\}$

b) $\sin x + \cos x = 1$

$\sin^2 x = 1 - 2\cos x + \cos^2 x$

$1 - \cos^2 x =$



$2\cos^2 x - 2\cos x = 0$

$2\cos x(\cos x - 1) = 0$

$x = \{0, \frac{\pi}{2}, \cancel{\frac{3\pi}{2}}\}$

CK

$x = 0 \quad 0 + 1 = 1$ ok

$x = \frac{\pi}{2} \quad 1 + 0 = 1$ ok

$x = \frac{3\pi}{2} \quad -1 + 0 = 1$ No

$x = \{0, \frac{\pi}{2}\}$

c) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{2} - \sin^{-1}(3x)$

$\frac{\pi}{6} = \frac{\pi}{2} - \sin^{-1}(3x)$

$\sin^{-1} 3x = \frac{2\pi}{3}$

$3x = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$x = \frac{\sqrt{3}}{6}$

d) $\sin 2x \tan^2 2x - \tan^2 2x = \sin 2x$

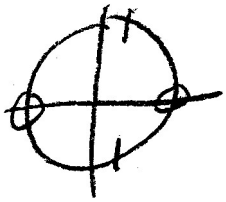
$\frac{\sin^3 2x \cos x}{\cos^2 2x} - \frac{\sin^2 2x}{\cos 2x} - \sin 2x = 0$

$\frac{\sin 2x}{\cos^2 2x}\left[\sin^2 2x - \cos 2x - \cos^2 2x\right] = 0$

$1 - \cos^2 2x$

$-\frac{\sin 2x}{\cos^2 2x}\left[2\cos^2 2x + \cos 2x - 1\right] = 0$

$(2\cos 2x + 1)(\cos 2x + 1)$



$\cos 2x = \frac{1}{2}$

$0 \le x < 2\pi$

$0 \le 2x < 4\pi$

$2x = \{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi, \frac{2\pi}{3}, 3\pi, \frac{11\pi}{3}\}$

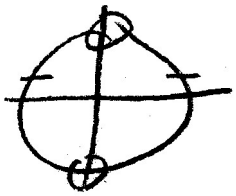
$x = \{0, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\}$

e) $4\cos x = 2\cos x \csc x$

$2\cos x(2 - \csc x) = 0$

$\csc x = 2$

$\sin x = \frac{1}{2}$



$x = \{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}\}$

II $\cos^{-1}x + \sin^{-1}x = K$

$$\cos(\cos^{-1}x + \sin^{-1}x) = \cos K$$

$$\cos\cos^{-1}x \cos(\sin^{-1}x) - \sin(\cos^{-1}x)\sin(\sin^{-1}x) = \cos K$$

$$x\sqrt{1-x^2} - \left(\sqrt{1-x^2}\right)x = \cos K$$

$$\cos K = 0 \quad \therefore K = \frac{\pi}{2}$$

III $\tan(\sec^{-1}x) \qquad |x| \geq 1$

$$\frac{\sin(\cos^{-1}\frac{1}{x})}{\cos(\cos^{-1}\frac{1}{x})}$$

$$\frac{\sqrt{1-\frac{1}{x^2}}}{\frac{1}{x}}$$

$$\frac{\frac{\sqrt{x^2-1}}{x}}{\frac{1}{x}}$$

$$\sqrt{x^2-1}$$

12/5/95

Rebecca wells 81
Ken TSAO 75
Shilah Rice 45
JANNA SWANK 34

Honors Trigonometry Name _____

Copy original problem. Per _____ Date _____

Convince *me* that you understand the concept!

Chapter 4 Make Up Exam

I Solve $\forall\ x\ \varepsilon\ [0,2\pi)$ (i.e. $0 \le x < 2\pi$).
Include "circle solutions" as well as a specific statement using proper set notation. (15 pts ea.)

a) $3\tan\frac{x}{2} = \cot\frac{x}{2}$

b) $\sin x + \cos x = 1$

c) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{2} - \sin^{-1}(3x)$

d) $\sin 2x \tan^2 2x - \tan 2x = \sin 2x$

e) $4\cos x = 2\cos x \csc x$

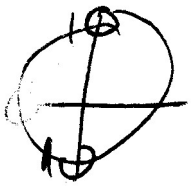
II Given: $\cos^{-1} x + \sin^{-1} x = k$. Determine the exact value of k. (15 pts)

III Determine $\tan\left(\sec^{-1} x\right)$. Be sure to describe restrictions on any variables, as usual. (10 pts)

I a) $\cos x - 2\sin^2 x + 2 = 0$

$\cos x - \cancel{2} + 2\cos^2 x \cancel{+2} = 0$

$\cos x\,(2\cos x + 1) = 0$



$x \in \{\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}\}$

b) $2 + 2\sin x = 4\cos^2 x$

$4 - 4\sin^2 x$

$4\sin^2 x + 2\sin x - 2 = 0$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

so $x = \frac{1}{2}$ $\sin x = -1$

$x \in \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\}$

c) $\sin^2 x = \sin x$

$\sin x(\sin x - 1) = 0$

$x \in \{0, \frac{\pi}{2}, \pi\}$

d) $\cot^2 x - \cos^2 x = \cot^2 x \cos^2 x$

$\cos^2 x\left(\frac{1}{\sin^2 x} - 1\right) = \frac{\cos^4 x}{\sin^2 x}$

$\cos^2 x\left[\left(\frac{1 - \sin^2 x}{}\right) - \cos^2 x\right] = 0$

$\cos^2 x\,(\cos^2 x - \cos^2 x] = 0$

$0 = 0$

$0 < x < 2\pi,\ x \neq \pi.$

e) $\sec^2 2x + \tan 2x = 1$

$s^2 + c^2 = 1$

$c^2 + 1 = \sec^2$

$\tan^2 2x + 1 + \tan 2x = 1$

$\tan 2x\,(\tan 2x + 1) = 0$

$2x \in \{0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}, 2\pi, \frac{11\pi}{4}, 3\pi, \frac{15\pi}{4}\}$

$x \in \{0, \frac{3\pi}{8}, \frac{\pi}{2}, \frac{7\pi}{8}, \pi, \frac{11\pi}{8}, \frac{3\pi}{2}, \frac{15\pi}{8}\}.$

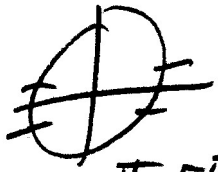
f) $\cot x \tan 2x = 3$

$\cot x \cdot \frac{2\tan x}{1 - \tan^2 x} = 3$

$2 = 3 - 3\tan^2 x$

$3\tan^2 x = 1$

$\tan x = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$



$x \in \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

II A) $\cos(\sin^{-1} u) = \sqrt{1-u^2}$

let $\sin^{-1} u = b$ $\quad |u| \le 1$ $\quad |b| \le \frac{\pi}{2}$

$u = \sin b$

$u^2 = \sin^2 b$

$u^2 = 1 - \cos^2 b$

$\cos^2 b = 1 - u^2$

$\cos b = \pm\sqrt{1-u^2}$

$\cos(\sin^{-1} u) = \sqrt{1-u^2}$

since b is 1st & 4th Quad. then $\cos b$ is > 0. Do not need $\pm$ sign.

II b) WTP $\csc^{-1}u = \sin^{-1}\left(\frac{1}{u}\right)$

let $\csc^{-1}u = b$ $\quad |u| \geq 1 \quad |b| \leq \frac{\pi}{2}, \; b \neq 0$

$u = \csc b$

$\frac{1}{u} = \sin b$

$\sin^{-1}\left(\frac{1}{u}\right) = b$

$= \csc^{-1}u$

c)

let $\cot^{-1}(u) = b$ $\quad u \in \mathbb{R}, \; 0 < b < \pi$

$u = \cot b$

$u \sin b = \cos b$

$u^2 \sin^2 b = \cos^2 b$

$u^2 \sin^2 b = 1 - \sin^2 b$

$\sin^2 b (1 + u^2) = 1$

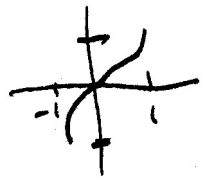
$\sin^2 b = \frac{1}{1+u^2}$

$\sin b = \frac{\pm 1}{\sqrt{1+u^2}}$

$\sin(\cot^{-1}(u)) = \frac{1}{\sqrt{1+u^2}}$

Since $b \in 1^{st}$ & 2nd quad. $\sin b > 0$ $\therefore$ no need of $\pm$ sign.

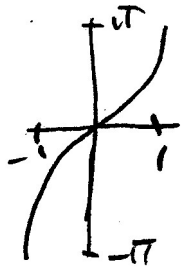
III a) $a(x) = \sin^{-1}x$



$D_a \; |x| \leq 1$

$R_a \; |a(x)| \leq \frac{\pi}{2}$

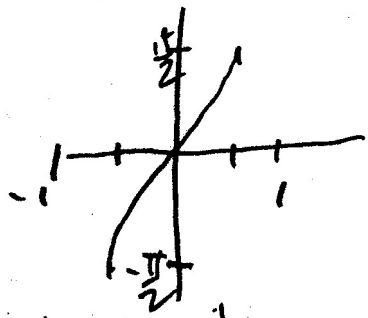
b) $b(x) = 2\sin^{-1}x$



$D_b \; |x| \leq 1$

$R_b \; |b(x)| \leq \pi$

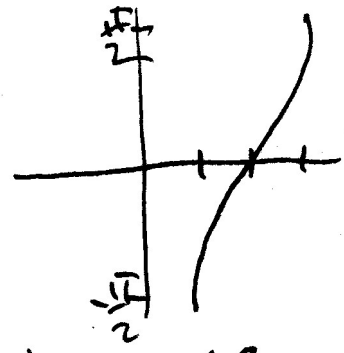
c) $c(x) = \sin^{-1}2x$



$D_c \; |x| \leq \frac{1}{2}$

$R_c \; |c(x)| \leq \frac{\pi}{2}$

D $d(x) = \sin^{-1}(x-2)$



$D_d \; 1 \leq x \leq 3$

$R_d \; |d(x)| \leq \frac{\pi}{2}$

12/3/97

Courtney Nicholas 92
Zac Hefner 87
Nicole Scarnell 86
Mick McCulley 84
Kristy French 82
Jerry Chmielewski 81

Honors Trigonometry

Name Trig Ch 4

Copy original problem.

Per _____ Date _____

Convince *me* that **you** understand the concept!

Make up Exam

Chapter 4 Exam

I Solve $\forall\ x\ \ 0 \le x < 2\pi$ **(10 pts ea)**

a) $\cos x - 2\sin^2 x + 2 = 0$

b) $2 + 2\sin x = 4\cos^2 x$

c) $\sin^2 x = \sin x$

d) $\cot^2 x - \cos^2 x = \cot^2 x \cos^2 x$

e) $\sec^2 2x + \tan 2x = 1$

f) $\cot x \tan 2x = 3$

II State ALL restrictions on "u" and on any other variables you may use. Justify *any* questionable (by me) steps. **(25 pts tot)**

a) Prove: $\cos(\sin^{-1} u) = \sqrt{1 - u^2}$

b) Prove: $\csc^{-1} u = \sin^{-1}\left(\frac{1}{u}\right)$

Type 1

c) Determine what: $\csc^{-1}(\cot^{-1} u)$ is equal to.
(hint: It might be something like the answer to part "a".)

III Draw the graph. State the domain and range. **(15 pts tot)**

a) $a(x) = \sin^{-1} x$

b) $b(x) = 2\sin^{-1} x$

c) $c(x) = \sin^{-1} 2x$

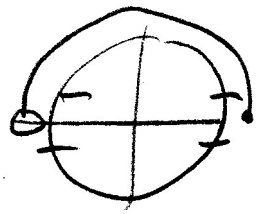
d) $d(x) = \sin^{-1}(x - 2)$

I

A) $3\tan\frac{x}{2} = \cot\frac{x}{2}$

$\tan^2\frac{x}{2} = \frac{1}{3}$

$\tan\frac{x}{2} = \pm\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$



$0 \le x < 2\pi$

$0 \le \frac{x}{2} < \pi$

$\frac{x}{2} \in \{\frac{\pi}{6}, \frac{5\pi}{6}\}$

$x \in \{\frac{\pi}{3}, \frac{5\pi}{3}\}$

B) $\sin x + \cos x = 1$

$\sin x = 1 - \cos x$

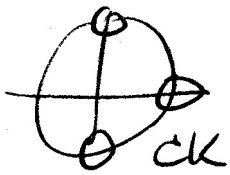
$\sin^2 x = 1 - 2\cos x + \cos^2 x$

$1 - \cos^2 x =$

$2\cos^2 x - 2\cos x = 0$

$2\cos x(\cos x - 1) = 0$

4pts



$x \in \{0, \frac{\pi}{2}, \cancel{\frac{3\pi}{2}}\}$

CK

$x = 0 \quad 0 + 1 = 1$ OK

$x = \frac{\pi}{2} \quad 1 + 0 = 1$ OK

$x = \frac{3\pi}{2} \quad -1 + 0 = 1$ NO

$x \in \{0, \frac{\pi}{2}\}$

C) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{2} - \sin^{-1}(3x)$

$\frac{\pi}{6} = \frac{\pi}{2} - \sin^{-1}3x$

$\frac{\pi}{3} = \sin^{-1}3x$

$\sin\frac{\pi}{3} = \sin(\sin^{-1}3x)$

$\frac{\sqrt{3}}{2} = 3x$

$x = \frac{\sqrt{3}}{6}$

D) $\sin 2x \tan^2 2x - \tan 2x = \sin 2x$

$\frac{\sin^3 2x}{\cos^2 2x} - \frac{\sin 2x}{\cos 2x} - \sin 2x = 0$

$\sin 2x\left(\frac{\sin^2 2x}{\cos^2 2x} - \frac{1}{\cos 2x} - 1\right) = 0$

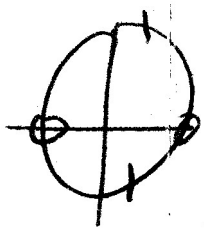
$\left(\frac{1 - \cos^2 2x}{\cos^2 2x}\right.$

$\sin 2x\,(1 - \cos^2 2x - \cos^2 x - \cos^2 2x) = 0$

$-\sin 2x\,(2\cos^2 2x + \cos 2x - 1) = 0$

$-\sin 2x\,(2\cos 2x - 1)(\cos 2x + 1) = 0$

$\cos 2x = \frac{1}{2} \qquad \cos 2x = -1$



$0 \le x < 2\pi$

$0 \le 2x < 4\pi$

$2x \in \{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi, \frac{7\pi}{3}, 3\pi, \frac{11\pi}{3}\}$

$x \in \{0, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\}$.

E) $4\cos x = 2\cos x \csc x$

$2\cos x(2 - \csc x) = 0$

$2 = \csc x$

$\sin x = \frac{1}{2}$

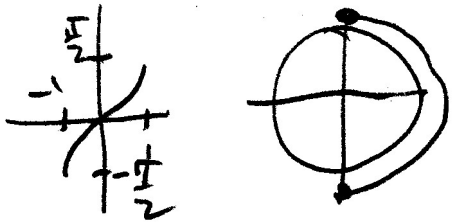
$x \{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}\}$

II

A) $A(x) = \sin^{-1} x$

D_A $-1 \le x \le 1$

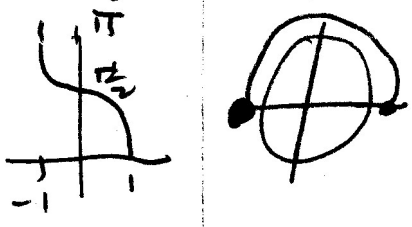
R_A $-\frac{\pi}{2} \le A(x) \le \frac{\pi}{2}$



B) $B(x) = \cos^{-1} x$

D_B $-1 \le x \le 1$

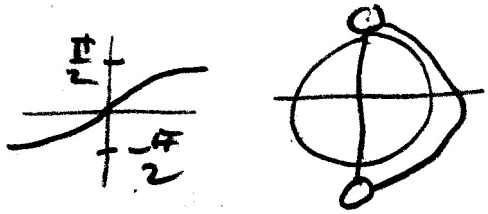
R_B $0 \le B(x) \le \pi$



C) $C(x) = \tan^{-1} x$

D_C $x \in \mathbb{R}$

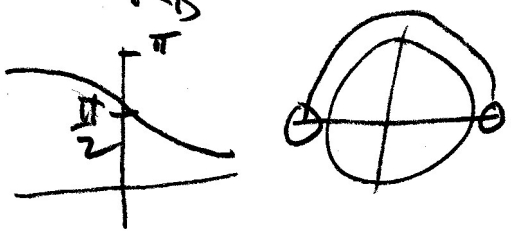
R_C $-\frac{\pi}{2} < C(x) < \frac{\pi}{2}$



D) $D(x) = \cot^{-1} x$

D_D $x \in \mathbb{R}$

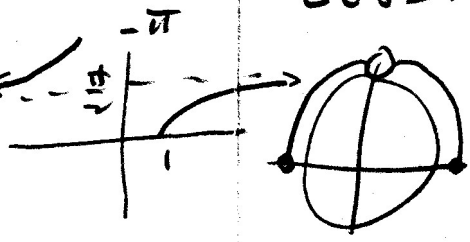
R_D $0 < D(x) < \pi$



E) $E(x) = \sec^{-1} x$

D_E $0 \le E(x) \le \pi$; $E(x) \ne \frac{\pi}{2}$

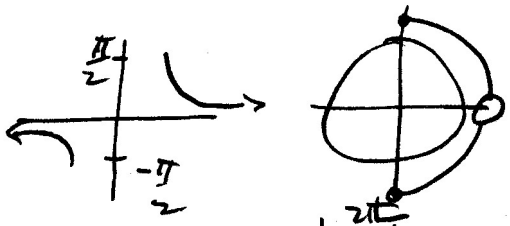
R_E $E(x) \le -1$ or $E(x) \ge 1$



$f(x) = \csc^{-1} x$

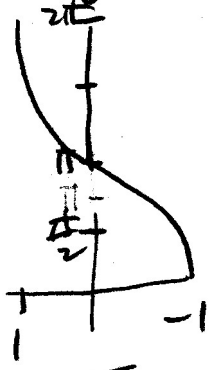
D_f $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

R_f $-\frac{\pi}{2} \le f(x) \le \frac{\pi}{2}$; $f(x) \ne 0$

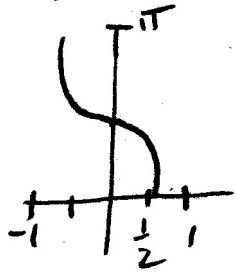


Extra

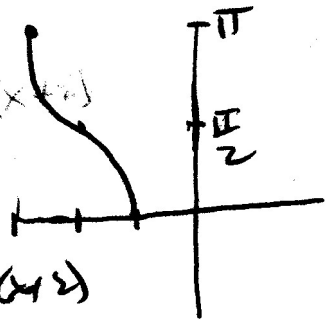
$a(x) = 2\cos^{-1} x$



$B(x) = \cos^{-1} 2x$



$C(x) = \cos^{-1}(x+2)$



$C(x) = \cos^{-1}(x+2)$

III

$\tan(\sec^{-1} x)$

$\tan(\cos^{-1}(\frac{1}{x})) =$

$\frac{\sin(\cos^{-1}(\frac{1}{x}))}{\cos(\cos^{-1}(\frac{1}{x}))} =$

$\frac{\sqrt{1 - \frac{1}{x^2}}}{\frac{1}{x}}$

$\frac{\frac{\sqrt{x^2-1}}{\pm x}}{\frac{1}{x}}$

$\pm\sqrt{x^2-1}$

OR

let $\sec^{-1} x = b$

$-1 \le x \le 1$

$0 \le b \le \pi$; $b \ne \frac{\pi}{2}$

$x = \sec b$

$x^2 = \sec^2 b$

$x^2 - 1 = \sec^2 b - 1$

$x^2 - 1 = \tan^2 b$

$\pm\sqrt{x^2-1} = \tan b$

$\pm\sqrt{x^2-1} = \tan(\sec^{-1} b)$

$s^2 + c^2 = 1$

$t^2 + 1 = \sec^2$

Tan is < 0 in Quad II so we Need the $\pm$ to make the $\sqrt{\ } < 0$. Keep the $\pm$ sign.

2 pts

12/14/98

100	1	1	2	105
90	0	1	1	
80	4	3	7	
70	3	8	11	
60	5	3	8	
59	7	9	16	
	67.7	66.8		7
	$\frac{1286}{19}$	$\frac{1671}{25}$	67.2	

9.3

	2	5	T	
100	0	0	0	98
90	4	5	9	
80	3	5	8	
70	5	3	8	
60	3	4	7	
59	9	7	16	
	$\frac{1615}{24}$ 67.3	$\frac{1643}{24}$ 68.5	67.9	15

Sect I right from freedom Rings.

Most copied and/or used past Student's work for freedom Rings

They paid The Price —

I will allow $\frac{1}{2}$ of Last Points but it will to a master piece to get a full half book!

I didn't allow pts Back in 98

Honors Trigonometry Name Trig Ch 4

Copy original problem. Per _____ Date 93/98

Convince *me* that you understand the concept!

No Calculators

Chapter 4 Exam

I

Solve $\forall\ x \in [0,2\pi)$ (i.e. $0 \le x < 2\pi$).
Use proper set notation and include "circle solutions" where appropriate. (15 pts ea.)

a) $3\tan\frac{x}{2} = \cot\frac{x}{2}$

#71

b) $\sin x + \cos x = 1$

#37

c) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{2} - \sin^{-1}(3x)$

76

d) $\sin 2x \tan^2 2x - \tan 2x = \sin 2x$

Problem 80

e) $4\cos x = 2\cos x \csc x$

II

For each of the following functions, state the domain and range, draw the graph of the function on a properly labeled axis and show the "memory device" we use for the function. (tot 15 pts)

a) $A(x) = \sin^{-1} x$ b) $B(x) = \cos^{-1} x$ c) $C(x) = \tan^{-1} x$

d) $D(x) = \cot^{-1} x$ e) $E(x) = \sec^{-1} x$ f) $F(x) = \csc^{-1} x$

True or false: $\tan(\sec^{-1} x) = \pm\sqrt{x^2-1}$

III

Determine $\tan(\sec^{-1} x)$. Be sure to describe restrictions on any variables, ~~as usual~~. You might use (10 pts)

Extra Credit ---------- **5 pts** ----------

Using the standard function $f(x) = \cos^{-1} x$ as a starting point,

sketch each of the following on separate axes:

a) $a(x) = 2\cos^{-1} x$ b) $b(x) = \cos^{-1} 2x$ c) $c(x) = \cos^{-1}(x+2)$

I

A) $-\sqrt{3}\sin x - \cos x = 1$

$-\sqrt{3}\sin x = 1 + \cos x$

$3\sin^2 x = 1 + 2\cos x + \cos^2 x$

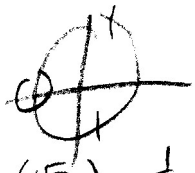
$3 - 3\cos^2 x =$

$4\cos^2 x + 2\cos x - 2 = 0$

$2(2\cos x - 1)(\cos x + 1) = 0$

$\cos x = \frac{1}{2} \quad \cos x = -1$

No check
-5



$(A+B)^2 = A^2 + B^2$
-5

$\frac{\pi}{3}$ $\quad -\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{2} = 1$

$-\frac{3}{2} - \frac{1}{2} = 1$

reject.

π $\quad -\sqrt{3}\cdot 0 - (-1) = 1$

$1 = 1$

$\frac{5\pi}{3}$ $\quad -\sqrt{3}\left(-\frac{\sqrt{3}}{2}\right) - \frac{1}{2} = 1$

$\frac{3}{2} - \frac{1}{2} = 1$

$1 = 1$

$x \in \{\pi, \frac{5\pi}{3}\}$

B) $\sin 2x \tan^2 2x - \tan 2x = \sin 2x$

$\sin 2x\left(\tan^2 2x - \frac{1}{\cos 2x} - 1\right) = 0$

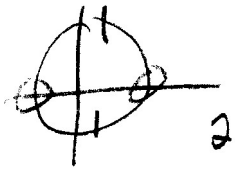
$\sin 2x\left(\frac{\sin^2 2x}{\cos^2 2x} - \frac{1}{\cos 2x} - 1\right) = 0$

$\frac{\sin 2x}{\cos^2 2x}\left(1 - \cos^2 2x - \cos 2x - \cos^2 2x\right) = 0$

$\frac{-\sin 2x}{\cos^2 2x}\left(2\cos^2 2x + \cos 2x - 1\right) = 0$

$(2\cos 2x - 1)(\cos 2x + 1)$

$\cos 2x = \frac{1}{2} \quad \cos 2x = -1$



$2x \in \{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi, \frac{7\pi}{3}, 3\pi, \frac{11\pi}{3}\}$

$x \in \{0, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\}$

C) $\tan\left(\pi + \sin^{-1}\frac{2}{3}\right) = x$

$\frac{\tan\pi + \tan\sin^{-1}\frac{2}{3}}{1 - \underbrace{\tan\pi}_{0}\tan\sin^{-1}\frac{2}{3}} = x$

$\sin\tan^{-1}u$
-3

$\frac{\sin\sin^{-1}\frac{2}{3}}{\cos\sin^{-1}\frac{2}{3}}$

$\frac{\frac{2}{3}}{\sqrt{1 - \frac{4}{9}}}$

$\frac{\frac{2}{\cancel{3}}}{\frac{\sqrt{5}}{\cancel{3}}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5} = x$

d) $\sec^2 x + \tan x = 1$

$s^2 + c^2 = 1$

$c^2 + 1 = \sec^2$

$\tan^2 x + 1 + \tan x = 1$

$\tan x(\tan x + 1) = 0$

$\tan x = 0 \quad \tan x = -1$

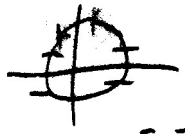
$x \in \{0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}\}$

e) $\sin^2 x + 5\cos^2 x - 4 = 0$

$1 - \cos^2 x + 5\cos^2 x - 4 = 0$

$4\cos^2 x = 3$

$\cos x = \pm \frac{\sqrt{3}}{2}$



$x \in \left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$

II

$\cot^{-1} x = \tan^{-1}\left(\frac{1}{x}\right) + \pi \quad x < 0$

Let $\cot^{-1} x = b \quad x < 0 \quad \frac{\pi}{2} < b < \pi$

$x = \cot b$

$\frac{1}{x} = \tan b$

$\tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}(\tan b)$

$= \tan^{-1}(\tan b - \pi)$

$= b - \pi$ since $-\frac{\pi}{2} < b - \pi < 0$

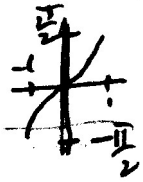
$\tan^{-1}\left(\frac{1}{x}\right) + \pi = \cot^{-1} x$ now in Range of $\tan^{-1} u$

III

$A(x) = \sin^{-1} x$

$D_A \quad |x| \le 1$

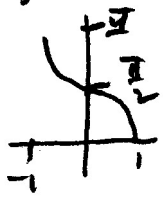
$R_A \quad |A(x)| \le \frac{\pi}{2}$



$B(x) = \cos^{-1} x$

$D_B \quad |x| \le 1$

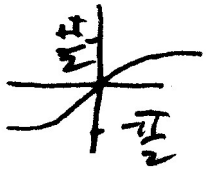
$R_B \quad 0 \le B(x) \le \pi$



$C(x) = \tan^{-1} x$

$D_C \quad x \in \mathbb{R}$

$R_C \quad -\frac{\pi}{2} < C(x) < \frac{\pi}{2}$



$D(x) = \cot^{-1} x$

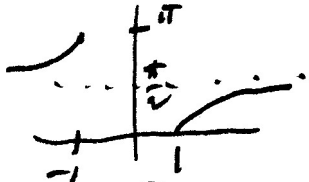
$D_D \quad x \in \mathbb{R}$

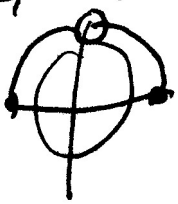
$R_D \quad 0 < D(x) < \pi$

$E(x) = \sec^{-1} x$

$D_E \quad |x| \ge 1$

$R_E \quad 0 \le E(x) \le \pi, \; E(x) \ne \frac{\pi}{2}$

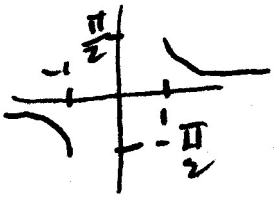


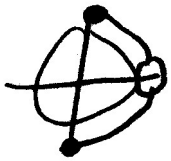


$f(x) = \csc^{-1} x$

$D_f \quad |x| \ge 1$

$R_f \quad |f(x)| < \frac{\pi}{2}, \; f(x) \ne 0$





Extra

$\sin^{-1}(\cos x) = 61$

$\cos x = \sin(90 - x)$

$= \sin 90 \cos x - \sin x \cos 90$

$= \cos x$

so

$\sin^{-1}(\cos x) = 61$

$\sin^{-1}(\sin 90 - x) = 61$

$90 - x = 61$

$x = 29$

	4	5	5
100	0	0	0
90	1	8	9
80	4	4	8
70	6	4	10
60	4	3	7
59	9	4	13

97

85

$\frac{1549}{24}$ $\frac{1995}{23}$ $\frac{3345}{47}$

64.5 78.1 71.2

Honors Trigonometry

Name Trig Ch 4

Per _____ Date 1999

Copy original problem.

Convince *me* that **you** understand the concept!

No Calculators, of course.

Chapter 4 Exam

use this on next AHS exam

I Solve $\forall\ x\ \varepsilon\ [0,2\pi)$ (i.e. $0 \le x < 2\pi$).
Use proper set notation and include "circle solutions" where appropriate. (15 pts ea.) 90

#39 a) $-\sqrt{3}\sin x - \cos x = 1$

#80 b) $\sin 2x \tan^2 2x - \tan 2x = \sin 2x$

I D 1998

#68 c) $\tan\left(\pi + \sin^{-1}\frac{2}{3}\right) = x$

#60 d) $\sec^2 x + \tan x = 1$

#7 e) $\sin^2 x + 5\cos^2 x - 4 = 0$

II Prove $\cot^{-1} x = \tan^{-1}\left(\frac{1}{x}\right) + \pi$ for $x < 0$ (15 pts)
Include explicit comments on each line explaining or justifying what you've done.

III For each of the following functions, state the domain and range, draw the graph of the function on a ***properly labeled axis*** and show the "memory device" we use for the function. (tot 10 pts)

a) $A(x) = \sin^{-1} x$ b) $B(x) = \cos^{-1} x$ c) $C(x) = \tan^{-1} x$

d) $D(x) = \cot^{-1} x$ e) $E(x) = \sec^{-1} x$ f) $F(x) = \csc^{-1} x$

Extra Credit ---------- 5 pts ----------

Solve for x where x is a degree measure : $\sin^{-1}(\cos x) = 61°$

"REEDOM" COVER SHEET

Name _____ Pg 5 of file

Per _____ Date _____

1) _____	2) _____	3) _____	4) _____
5) _____	6) _____	7) _____	8) _____
9) _____	10) _____	11) _____	12) _____
13) _____	14) _____	15) _____	16) _____
17) _____	18) _____	19) _____	20) _____
21) _____	22) _____	23) _____	24) _____
25) _____	26) _____	27) _____	28) _____
29) _____	30) _____	31) _____	32) _____
33) _____	34) _____	35) _____	36) _____
37) _____	38) _____	39) _____	40) _____
41) _____	42) _____	43) _____	44) _____
45) _____	46) _____	47) _____	48) _____
49) _____	50) _____	51) _____	52) _____
53) _____	54) _____	55) _____	56) _____
57) _____	58) _____	59) _____	60) _____
61) _____	62) _____	63) _____	64) _____
65) _____	66) _____	67) _____	68) _____
69) _____	70) _____	71) _____	72) _____
73) _____	74) _____	75) _____	76) _____
77) _____	78) _____	79) _____	80) _____
81) _____	82) _____	83) _____	84) _____
85) _____	86) _____	87) _____	88) _____
89) _____	90) _____	91) _____	92) _____
93) _____			

"FREEDOM" 95
COVER SHEET

Name _____ Per _____ Date _____

1) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$
2) $\frac{\pi}{2}$
3) $\frac{\pi}{2}\ \frac{3\pi}{2}$
4) $0\ \pi\ \{0\ \frac{\pi}{2}$
5) $\frac{\pi}{4}\ \frac{\pi}{2}\ \frac{5\pi}{4}\ \frac{3\pi}{2}$
6) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$
7) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$
8) ϕ
9) $\frac{\pi}{3}\ \frac{2\pi}{3}\ \frac{4\pi}{3}\ \frac{5\pi}{3}$
10) 0
11) $\frac{3\pi}{2}$
12) $\frac{\pi}{3}\ \frac{\pi}{2}\ \frac{3\pi}{2}\ \frac{5\pi}{3}$
13) $0\ \pi$
14) $0\ \pi$
15) $\frac{2\pi}{3}\ \frac{4\pi}{3}$
16) $0\ \frac{\pi}{3}\ \frac{2\pi}{3}\ \pi$
17) π
18) $0\ \frac{\pi}{3}\ \pi\ \frac{5\pi}{3}$
19) $\frac{\pi}{3}\ \pi\ \frac{5\pi}{3}$
20) $0\ \frac{\pi}{2}\ \pi$
21) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$
22) $\frac{\pi}{2}\ \frac{3\pi}{2}$
23) $0\ \frac{3\pi}{2}$
24) $0\ \pi\ \frac{7\pi}{6}\ \frac{11\pi}{6}$
25) $\frac{\pi}{6}\ \frac{\pi}{2}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$
26) ϕ
27) $\frac{\pi}{4}\ \frac{3\pi}{4}\ \frac{5\pi}{4}\ \frac{7\pi}{4}$
28) $0\ \pi\ \{0\ \frac{\pi}{2}$
29) $0\ \pi$
30) $\frac{5\pi}{6}\ \frac{11\pi}{6}\ \{\frac{\pi}{6}\ \frac{7\pi}{6}\}$
31) $\frac{7\pi}{6}\ \frac{11\pi}{6}$
32) $\frac{\pi}{3}\ \frac{\pi}{2}\ \frac{3\pi}{2}\ \frac{5\pi}{3}\ \frac{\pi}{2}\ \frac{2\pi}{3}\ \frac{4\pi}{3}\ \frac{3\pi}{2}$
33) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$
34) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$
35) $\frac{\pi}{4}\ \frac{5\pi}{4}$
36) $\frac{\pi}{4}\ \frac{3\pi}{4}\ \frac{5\pi}{4}\ \frac{7\pi}{4}$
$2+\sqrt{3}$
37) $0\ \frac{\pi}{2}$
38) ϕ
39) $\pi\ \frac{5\pi}{3}$
40) $\frac{\pi}{3}\ \pi\ \frac{5\pi}{3}$
41) $0\ \frac{\pi}{4}\ \pi\ \frac{5\pi}{4}$
42) $0\ \pi$
43) $\frac{5\pi}{24}\ \frac{11\pi}{24}\ \frac{29\pi}{24}\ \frac{35\pi}{24}$
44) $0\ \frac{\pi}{6}\ \frac{5\pi}{6}\ \pi$
46) $\frac{\pi}{6}\ \frac{5\pi}{6}$ $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$
47) $\frac{\pi}{2}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$
48) $\frac{\pi}{2}\ \frac{3\pi}{4}\ \frac{3\pi}{2}\ \frac{7\pi}{4}$
$\frac{7\pi}{6}\ \frac{11\pi}{6}$
49) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$
50) $\frac{\pi}{2}\ \frac{5\pi}{6}\ \frac{3\pi}{2}\ \frac{11\pi}{6}$
51) $0\ \frac{2\pi}{3}\ \frac{4\pi}{3}$
52) $\frac{2\pi}{3}\ \pi\ \frac{4\pi}{3}$
53) $\frac{\pi}{6}\ \frac{5\pi}{6}$
54) $\frac{\pi}{2}$
55) $\{\frac{\pi}{2}\ \frac{3\pi}{2}\}$
56) $\frac{\pi}{4}\ \frac{7\pi}{4}$
57) $\frac{\pi}{6}\ \frac{\pi}{2}\ \frac{5\pi}{6}$
57) $\frac{1}{2}$
58) ϕ
59) $\{\frac{\pi}{2}\}$
60) $\frac{\pi}{2}\ \frac{3\pi}{2}$
$0\ \frac{3\pi}{8}\ \frac{\pi}{2}\ \frac{7\pi}{8}\ \frac{11\pi}{8}\ \frac{3\pi}{2}\ \frac{15}{8}$
61) $0 \le x < 2\pi$
62) $\frac{7\pi}{6}\ \frac{11\pi}{6}$
63) $\{\frac{\pi}{2}\ \frac{7\pi}{6}\ \frac{11\pi}{6}\}$
64) 0
$0\ \frac{\pi}{4}\ \frac{\pi}{2}\ \frac{5\pi}{4}\ \frac{3\pi}{2}$
65) $\pi,\ \frac{7\pi}{6}\ \frac{11\pi}{6}$
66) $\frac{\pi}{6}\ \frac{\pi}{2}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$
67) $\frac{\pi}{6}\ \frac{3\pi}{4}\ \frac{5\pi}{6}\ \frac{7\pi}{4}$
68) $\frac{2\sqrt{5}}{5}$
69) $\frac{\pi}{2}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$
70) 0
71) $\frac{\pi}{3}\ \frac{5\pi}{3}$
72) $\frac{2\pi}{3}\ \frac{4\pi}{3}$
73) $2\ \{-2\}$
74) $\frac{\pi}{10}\ \frac{\pi}{2}\ \frac{9\pi}{10}\ \frac{13\pi}{10}\ \frac{17\pi}{10}$
75) $\frac{\pi}{12}\ \frac{\pi}{4}\ \frac{5\pi}{12}\ \frac{7\pi}{12}\ \frac{3\pi}{4}\ \frac{11\pi}{12}$
$\frac{13\pi}{12}\ \frac{5\pi}{4}\ \frac{17\pi}{12}\ \frac{19\pi}{12}\ \frac{7\pi}{4}\ \frac{23\pi}{12}$
76) $\frac{\sqrt{3}}{6}$
77) $\pm\frac{3\sqrt{10}}{10}$
78) $\frac{\pi}{12}\ \frac{11\pi}{12}\ \frac{13\pi}{12}\ \frac{23\pi}{12}$
79) $\frac{5\sqrt{3}+12}{26}$
80) $0\ \frac{\pi}{6}\ \frac{\pi}{2}\ \frac{5\pi}{6}\ \pi\ \frac{7\pi}{6}\ \frac{3\pi}{2}$
$\frac{11\pi}{6}$
81) $\pm\sqrt{\frac{34-5\sqrt{34}}{68}}$
82) $0 \le x < 2\pi\ \ x \ne \pi$
83) $\frac{\pi}{2}\ \frac{3\pi}{2}$
84) $\frac{\pi}{3}\ \frac{\pi}{2}\ \frac{2\pi}{3}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{4\pi}{3}\ \frac{5\pi}{3}\ \frac{11\pi}{6}$
$\frac{\pi}{9}\ \frac{2\pi}{9}\ \frac{4\pi}{9}\ \frac{5\pi}{9}\ \frac{7\pi}{9}\ \frac{8\pi}{9}\ \frac{10\pi}{9}\ \frac{11\pi}{9}\ \frac{13\pi}{9}\ \frac{14\pi}{9}\ \frac{16\pi}{9}\ \frac{17\pi}{9}$
88) $\frac{9}{4},\ 4$
89) $0 \le x < 2\pi\ \ x \ne \{\frac{\pi}{2}\ \frac{3\pi}{2}\}$
90) $0\ \frac{\pi}{3}\ \frac{2\pi}{3}\ \pi$
91) 16
92) $0 \le x < 2\pi\ \ x \ne \{\frac{\pi}{2}\ \frac{3\pi}{2}$
93) $\frac{3\pi}{10}\ \frac{7\pi}{10}\ \frac{3\pi}{2}\ \frac{19\pi}{10}$

4 28 30 32 35
46 54 58 62 73

"FREEDOM RINGS"

There are 76 required problems. Total value is 10 assignments of "+." Each error or omission reduces the 10 by 1 level. (Total 10 wrong = 10 checks; total 15 wrong = 5 checks, 5 zeros; etc.) An error in recording your answer on the answer sheet *is* an error. An answer on the answer sheet without the proper work is an error on the answer sheet *and* an error on the work! (i.e. 2 errors). There are 17 "extra" problems. These will forgive up to 6 errors.

Some problems will have obvious methods; other will (multiple angle problems, for example) need an explanation justifying method. When in doubt, explain. The work **must** be neat, in order, and *easy* to read. Comments and/or explanations are to be adjacent to their problem. Each page is to have your name and period number in the upper right hand corner. After a problem is solved, write the *answer only* (not in set notation) on the cover sheet. The complete answer, in proper notation is to be at the bottom of its problem.

Solve for x as an element of the reals or for all $x \in [0, 2\pi)$ where appropriate. In the case of "inverse" trig functions, your solution is to be with the proper range and/or domain of that particular function. Include "circle solutions" in addition to proper set notation where appropriate.

Fill in the cover page with your name, period number and numbered 1 through 93 with the answers only to each problem OR leave a blank if you could not do the problem. Attach this cover sheet on the top of all your work when you are ready to turn it in for grading. The entire packet is due the day the Chapter 4 exam is given (according to the HW sked). The packet is due when you enter the room, **NOT** after you take the exam. Late work begins with 10 errors.

These problems and their proper solutions are important. Check with fellow students to verify that you (or they) have not made costly errors. There is no partial credit on any problem. Any error, not matter how insignificant, is still an error and costs points.

When you have completed this task, you will be **free** of any doubts concerning your ability to use algebra, your knowledge of trigonometric identities, and your prowess with inverse trig functions! You have joined many patriots before you. Paul Revere, American patriot, skilled craftsman, silversmith, engraver, and printer, (before he made cooking pots and pans called *Revereware*) celebrated completion of his homework with a ride warning Massachusetts colonists that British troops were marching toward Lexington and Concord to seize militia supplies and that he had completed **Freedom Rings**. He was immortalized in a Longfellow poem. The facts about his math work had been lost until now.

1) $2\sin^2 x - 1 = -\sin x$
2) $2\cos^2 x = \sin x - 1$
3) $\sin^2 x = 1$
4) $\sin x = \tan x$
5) $\cot^2 x = \cot x$
6) $\cot x \tan 2x = 3$
7) $4\cos^2 x - 3 = 0$
8) $(3\sec x - 2)(\cos x) = 0$
9) $\cos^2 x = \frac{1}{4}$
10) $\tan^{-1}(1 + x) + \tan^{-1}(1 - x) = \frac{\pi}{2}$
11) $2\sin^2 x - \sin x = 3$
12) $2\cos^2 x = \cos x$
13) $\sin 2x = 2\sin x$
14) $\cos x \tan x = 3\tan x$
15) $2\cos x + 1 = 0$
16) $2\sin^2 x - \sqrt{3}\sin x = 0$
17) $2\sin^2 x + \cos x = -\cos^2 x$
18) $2\sin x \cos x = \sin x$
19) $2\sin^2 x - \cos x = 1$
20) $\sin^2 x = \sin x$
21) $\cos 2x = \sin x$
22) $\sin^2 \frac{x}{2} + \tan^2 \frac{x}{2} = \frac{3}{2}$
23) $\sin x \cos x + \cos x = 1 + \sin x$
24) $2\sin^2 x = -\sin x$
25) $\sin 2x = \cos x$
26) $2\sin x = 3$
27) $\tan x = \cot x$
28) $\sin x + \sec^2 x = 1 + \sin^2 x + \tan^2 x$
29) $\cos^2 x - \sin^2 x = 1$
30) $\sqrt{3}\tan x = -1$
31) $2\sin x + 1 = 0$
32) $\cos x - 2\sin^2 x + 2 = 0$
33) $2 + 2\sin x = 4\cos^2 x$
34) $\sec^2 x = \frac{4}{3}$
35) $3\tan x - 4 = \tan x - 2$
36) $\arctan(x - 2) = \frac{\pi}{3}$

37) $\sin x + \cos x = 1$

38) $\tan\left(\frac{\pi}{4} + 3x\right) = 2\tan 6x$

39) $-\sqrt{3}\sin x - \cos x = 1$

40) $\sin 2x \sin x - \cos 2x \cos x = 1$

41) $\sin 2x + \cos 2x = 1$

42) $2\cos^2 x = 1 + \cos^2 x$

43) $\sin^2\left(x + \frac{\pi}{6}\right) - \cos^2\left(x + \frac{\pi}{6}\right) = \frac{\sqrt{2}}{2}$

44) $\cos 2x + \sin x = 1$

45) $2\sin^2 x + 3\sin x - 2 = 0$

46) $2\sin^2 x - \sin x - 1 = 0$

47) $2\cot^2 x + 2\cot x = 0$

48) $\csc^2 x + 2\csc x = 0$

49) $\tan 2x \tan x = 1$

50) $\cos^2\left(x + \frac{\pi}{3}\right) - \sin^2\left(x + \frac{\pi}{3}\right) = \frac{1}{2}$

51) $\cos 2x + 2\sin^2 \frac{x}{2} = 1$

52) $2\cos^2 x + 3\cos x = -1$

53) $2\sin x \sec x = \sec x$

54) $\sin^2 x - 2\sin x + 1 = 0$

55) $2\sec x - 2\sqrt{2} = 0$

56) $2\sin^2 x - 3\sin x + 1 = 0$

57) $\cos^{-1} 2x = \frac{\pi}{3} - \cos^{-1} x$

58) $\tan^{-1} x + \tan^{-1} 1 = \frac{3\pi}{4}$

59) $2\sin\left(x - \frac{\pi}{6}\right) = \sqrt{3}\sin x$

60) $\sec^2 2x + \tan 2x = 1$

61) $\sin^2\left(x + \frac{\pi}{2}\right) + \cos^2\left(x + \frac{\pi}{2}\right) - 1 = 0$

62) $2\cos 2x + \sin 2x \sec x = 0$

63) $3\sec x - \cos x = 2$

64) $2\cos^2 x \sin^2 x - \sin x \cos x = 0$

65) $\sin 2x + \cos x + 2\sin x = -1$

66) $\sin(\pi - 2x) = \cos x$

67) $2\tan x \sin x + 2\sin x = \tan x + 1$

68) $\tan\left(\pi + \arcsin \frac{2}{3}\right) = x$

69) $\sin x = \cos(2x - \pi)$

70) $\sin 2x \sin x + \cos x \cos 2x = 1$

71) $3\tan \frac{x}{2} = \cot \frac{x}{2}$

72) $2 + \sec x = 0$

73) $\tan\left(\tan^{-1} x + \tan^{-1} 1\right) = -3$

74) $\sin 2x \cos 3x + \cos 2x \sin 3x = 1$

75) $\tan^2 3x = 1$

76) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{2} - \sin^{-1} 3x$

77) $\cos\left(\frac{1}{2}\arctan \frac{3}{4}\right) = x$

78) $2\cos 2x = \sqrt{3}$

79) $\cos\left(\arcsin \frac{-1}{2} + \cos^{-1} \frac{5}{13}\right) = x$

80) $\sin 2x \tan^2 2x - \tan 2x = \sin 2x$

81) $\sin\left(\frac{1}{2}\arctan \frac{3}{5}\right) = x$

82) $\cot^2 x - \cos^2 x = \cot^2 x \cos^2 x$

83) $2\cos x = 4\cos x \csc x$

84) $\tan 2x = 3\cot 2x$

85) $\cos^3 x - \sin^3 x = (\cos x - \sin x)\left(1 + \frac{1}{2}\sin 2x\right)$

86) $\cos x = \cos 2x$

87) $\tan^2 3x = 3$

88) $\sec^{-1} \frac{5}{3} + \csc^{-1} \frac{5}{4} = 2\tan^{-1} \frac{x}{3}$

89) $\tan^2 x - \sec^2 x = -1$

90) $\sqrt{3}\tan x = 2\sin x \tan x$

91) $\sin^{-1} \frac{3}{5} + \sin^{-1} \frac{12}{13} = \cos^{-1} \frac{-x}{65}$

92) $1 - 2\sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$

93) $\sin^3 5x = -1$